A C T A U N I V E R S I T A T I S L O D Z I E N S I S
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THE INFLUENCE OF CHOICE OF METHODS OF CHECKING ON TOTAL QUALITY CONTROL COSTS

Abstract. Acceptance sampling is commonly used in factories, because it decreases the probability of giving defective batch to the customer. From two distinguished ways of acceptance sampling, more frequently in practice is used an attribute sampling method. The purpose of this article is to emphasize the benefits of using acceptance sampling by variables.
Key words: Statistical quality control, acceptance sampling, acceptance sampling costs.

I. INTRODUCTION

Factories, in order to decrease probability of giving defective batch of products to the customer, use acceptance control. The most frequently, this control is based on attribute estimation, because it is less complicated than estimation by variables, and many times it is the only possible way to control quality of elements. Nevertheless acceptance sampling by variables has many advantages. It is common to regard, that it makes lower total quality control costs, because of smaller size of samples. (Steczkowski, 1993). The purpose of this article is exploration, if lower size of sample, in case of acceptance sampling by variable, indeed causes that this way of quality control makes lower costs than control based on attribute estimation. The hypothesis, which says, that the acceptances sampling by variables makes lower total quality control costs than the attribute acceptance sampling, is assumed.

II. DESCRIPTION OF ACCEPTANCE SAMPLING PLANS

Acceptance sampling plan is the procedure of deciding the quality of batch on the basis of the sample, which is randomly selected from this batch. It with regard of the way of conduction examination, may impart to attribute acceptance sampling plans and attribute acceptance plans by variables. The first of mentioned acceptance

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sampling plan groups is used, when the elements, which are controlled, are classified as "good" or "wrong". This kind of control is applied especially to estimation products with regard to unmeasurable features, because in this case it is the only possible way to control the quality of elements.

The acceptance sampling plans by variable are able to apply only in case of measurable features of product. These plans are based on measuring average and variability of examined feature. The examined feature is assumed to have a normal distribution which mean μ and standard deviation σ (Wetherill, Chiu, 1975). The acceptance sampling plans by variable may be build in case of σ is known, as well as in case of σ is unknown. There are also assumed acceptance sampling plans for one – sided and two – sided specification limits. This condition for one – sided, lower specification limit L, where k is an acceptance constant, in case of known σ, is presented by the formula:

$$\frac{\bar{x} - L}{\sigma} \geq k \tag{1}$$

The characteristic of acceptance sampling plans is Lot Tollerance Percent Defective. It is such true defectiveness of batch, for which acceptance of the batch by given acceptance sampling plan application is executed with assumed lower probability β. Acceptance Quality Level (AQL) is such true defectiveness of batch, for which acceptance of the batch by given acceptance sampling plan application is executed with assumed high probability $1-\alpha$.

III. COST OF QUALITY CONTROL

The concept of quality control costs means all enterprise's costs connected with conduction quality control. Among costs of statistical quality control, there were distinguished costs of prevention, costs of examination and estimation, loss of internal lack and loss of external lack (Iwasiewicz, 1999). Total quality control cost is a sum of all distinguished groups of costs.

The total acceptance control cost consists of costs of examination and estimation, loss because of internal and external lack. The costs of examination and estimation depend on single costs of control (k_k) and on amount of controlled pieces. To signify as *N*- size of batch, *n* - size of sample, *D* - amount of defective elements in batch, *d* - amount of defective elements in sample, control cost by application the single – stage acceptance sampling are:

$$K_k(p) = L(p)nk_k + \big(1 - L(p)\big)Nk_k \tag{2}$$

Loss of internal lack, depends on cost of exchange (or repair) of defective element $\left(k_{bw}\right)$ and on amount of damage elements detected in sample, in case of acceptance batch, or on amount of damage elements detected in batch, in case of its rejection (Kończak, 2007):

$$K_{bw}(p) = L(p)\sum_{d=0}^{c}\frac{\binom{D}{d}\cdot\binom{N-n}{n-d}}{\binom{N}{n}}dk_{bw} + \big(1 - L(p)\big)Dk_{bw} \tag{3}$$

Loss of external lack is found only in situation, when the batch is accepted. This cost embraces the guarantee repairs and loss as a results of forfeiture customers and reputation of company. To assume, that single cost resultant on external loss is equal k_{bz}, this cost is equal:

$$K_{bz}(p) = k_{bz}\sum_{k=0}^{c}\big(D-k\big)\cdot P\big(X=k\big) \tag{4}$$

IV. SIMULATION ANALYSES

To exploration, if lower size of sample, in case of acceptance sampling by variable, indeed causes that this way of quality control makes lower costs than control based on attribute estimation, number of computer simulations were carried out. On the necessity of these simulations it was established, that controlled variable have a normal distribution with parameters with μ and σ equal adequately 1[u] and 0,005 [u]. It was also assumed that the number of batches are *N*={132,200,250,300,…,1000} pieces. As lower specification limit assumed 0,99 [u]. It is aimed at the probability of acceptance of batch, which defectiveness is equal 0,001 amount to 0,95, and probability of rejection of batch, which defectiveness is equal 0,05 amount to 0,90. An acceptance sampling plan, which meets given assumptions independently of size of sample, is the plan *P*(132;3) for attribute estimation and the plan *P*(19; 1,95) for estimation by variable. In the analyses there are also considered two systems of distribution of weight of costs. In the A system it was assumed, that the single cost of element control is equal 4[u], cost of internal lack is 8[u], and the cost of external lack reaches 15 [u]. In the B system it is adequately assumed 1, 10, 30 units for particular components

of total quality control costs. In the analyses there was also established, that the single cost of control elements in case of acceptance control by variable, is about 20% higher than in case of attribute acceptance control, independently on adopted distribution of weight of costs.

Table 1. Total quality control costs for attribute and by variable acceptance sampling plans (A system)

	Attribute acceptance sampling				Acceptance sampling by variable			
N	Costs of examination and estimation	Loss because of internal lack	Loss because of external lack	Total costs	Costs of examination and estimation	Loss because of internal lack	Loss because of external lack	Total costs
132	528,00	24,21	0,00	552,21	317,60	12,11	22,69	352,40
200	625,51	28,46	14,80	668,77	445,41	16,86	36,55	498,82
400	905,45	41,17	59,52	1006,14	847,04	32,23	76,29	955,56
600	1193,87	54,36	103,20	1351,43	1221,78	46,49	117,96	1386,23
800	1454,65	65,98	149,75	1670,37	1644,70	62,64	156,01	1863,35
1000	1789,38	81,99	188,76	2060,13	2056,18	78,38	195,54	2330,09

Source: own elaboration.

The procedure of simulation way of appointing of total quality control cost for attribute acceptance sampling plans, as well as for acceptance sampling plans by variable proceeded by stages. In the first step the parameters of acceptance plans, which are characterized by given level of consumer and producer risk, were appointed. Next the N – elements batch was generated (in base on normal distribution with parameters μ and σ). In the third step the samples were taken and their control was happened. This batch, which have met assumptions proper for given way of acceptance control was accepted. Next, the particular quality control costs were calculated. The costs of estimation are product of single control cost and amount of controlled elements, what mean amount of elements, which create sample, if the batch was accepted and amount of elements, which constitute batch, if it was rejected. The loss of internal lack are product of amount of defective elements, which were detected in sample, if the batch was accepted or of amount of lacks, which were in batch, if it was rejected and of single cost of internal lack. The loss of external lack are product of amount of single cost of external lack and of defective elements, which were passed to the customer. The whole procedure was repeated 1000 times and the results were averaged.

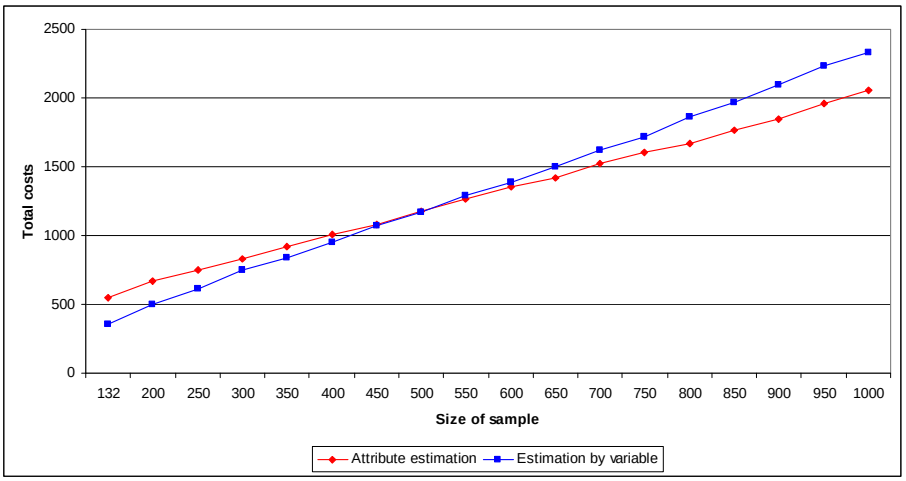

Figure 1. Total quality control costs for attribute and by variable acceptance sampling plans (A system)

Source: own elaboration.

Table 1 and figure 1 present the formation of total quality control costs for attribute and by variable acceptance sampling plans depending on the size of batch in case of A systems. From analyses of them there comes the conclusion, that for samples, which are small in number it is more advantageous to apply acceptance sampling plan by variables, and for samples, which are in big number, smaller costs make attribute acceptance sampling.

Table 2. Total quality control costs for attribute and by variable acceptance sampling plans (B system)

	Attribute acceptance sampling				Acceptance sampling by variable			
N	Costs of examination and estimation	Loss because of internal lack	Loss because of external lack	Total costs	Costs of examination and estimation	Loss because of internal lack	Loss because of external lack	Total osts
132	132,00	29,99	0,00	161,99	78,90	14,98	45,05	138,92
200	156,80	36,00	29,96	222,76	111,20	21,25	74,21	206,66
400	226,28	51,34	117,71	395,33	210,34	39,72	152,57	402,64
600	299,73	68,46	205,42	573,61	314,44	59,56	232,11	606,11
800	367,40	83,42	294,50	745,33	403,68	76,25	316,02	795,95
1000	441,27	100,61	383,88	925,75	509,34	96,90	395,00	1001,23

Source: own elaboration.

The relation between total quality control costs for both ways of control comes out of changing structure of this cost. The total quality control cost consists almost exclusively of cost of examination and estimation, however, when the size of batch increases, the participation in general cost of costs of lacks, especially of external lacks, also increases.

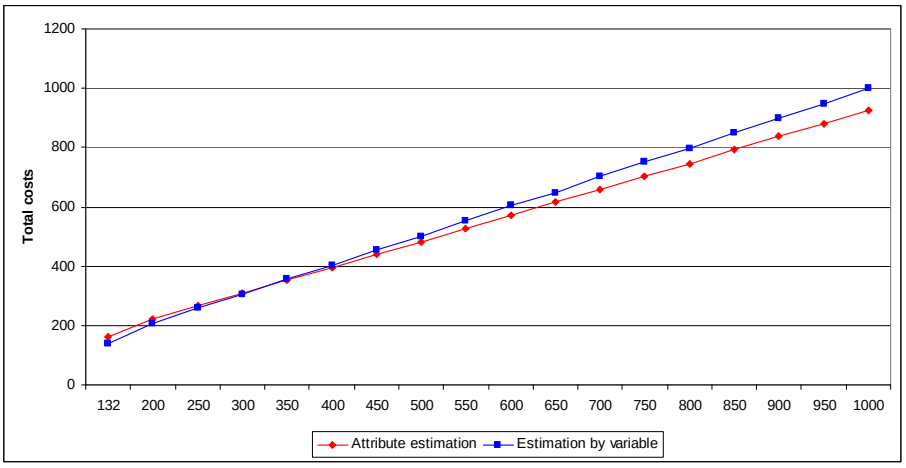

Figure 2. Total quality control costs for attribute and by variable acceptance sampling plans (B system)

Source: own elaboration.

The table 2 and figure 2 presents the formation of total quality control costs for analysed ways of quality control depending on the size of batch in case of B systems. In this case, even for batch, with its small size, it is more advantageous to apply attribute acceptance sampling plan. It mainly comes out of the fact, that during control, major amount of elements, is able to detect major amount of defects, so less damage elements are donated to customer. It is necessary to remember that, with all elements like this, which are donated to customer, loss are born. Significant part of total quality control cost, in this case constitute costs of estimation, but big share of total cost has also costs of external lacks.

Table 3. Total quality control costs for attribute and by variable acceptance sampling plans for batch consists of mixture of elements with value of analyzed feature, which comes from different distributions

System of weight of costs	Number of sumple	200		500		1000	
	Acceptance sampling	Attribute	By variable	Attribute	By variable	Attribute	By variable
A	Costs of examination and estimation	644,47	480,16	1163,02	1139,63	2010,54	2209,22
	Loss because of internal lack	38,41	30,34	80,94	74,31	150,27	146,49
	Loss because of external lack	9,84	24,96	53,62	66,06	126,55	133,63
B	Costs of examination and estimation	160,44	114,00	287,48	258,93	497,78	500,86
	Loss because of internal lack	39,67	23,29	71,79	53,72	124,81	104,58
	Loss because of external lack	29,35	78,47	158,88	213,10	375,50	436,19

Source: own elaboration.

The consideration brought up to this time concerned case, when the examined feature have a normal distribution with parameters agreeable with fixed norms. In practice the parameters of observed feature are no all time the same. Let's assume, that batch is mixture of elements with value of analyzed feature, which comes from normal distribution with parameters μ and σ and of elements with value of analyzed feature, which comes from normal distribution with parameters μ and $\sigma + k$, where $k \in \langle -0{,}5 \cdot \sigma; 0{,}5 \cdot \sigma \rangle$.

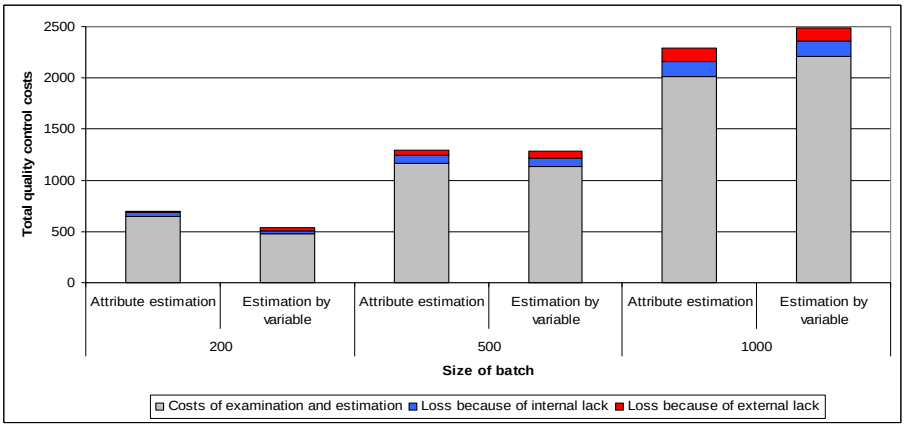

Figure 3. The structure of total quality control cost for attribute and by variable acceptance sampling (A system)

Source: own elaboration.

The table 3 and the figures 3 and 4 present the formation of total quality control costs for analyzed ways of quality control for three selected sizes of batch in case of A and B systems of weigh of costs. The figures reflected also the structure of quality costs in analyzed cases. On the basis of them, we may come to conclusion that in case of mixture of elements with value of analyzed feature, which comes from different distribution, acceptance sampling plan by variable achieves better results than in case of the situation when all elements of batch are characterized by the value of analyzed feature, which comes from normal distribution with parameters agreeable with fixed norms. It is necessary to observe, that for two of analyzed numbers of batch, this plan makes lower total quality control costs than attribute sampling plan. From analyses of those figures there comes also a conclusion, that the structure of quality control costs is less sensitive on appearing in batch mixture of elements with value of analyzed feature, which comes from different distributions.

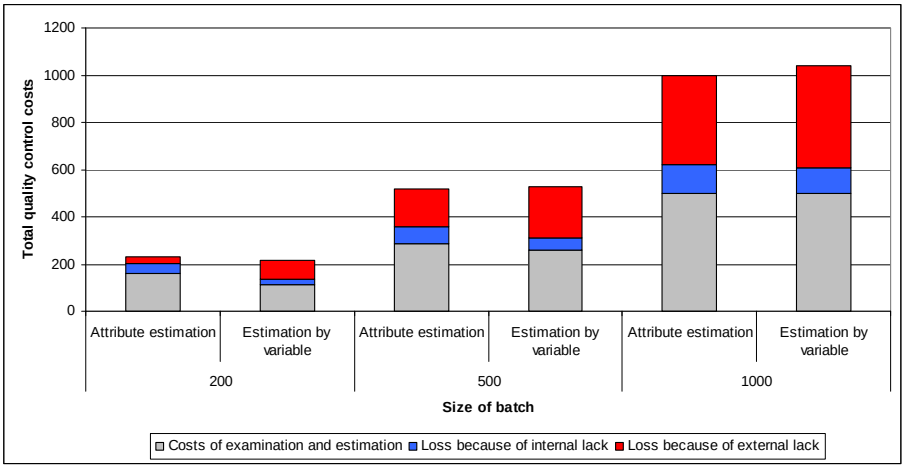

Figure 4. The structure of total quality control cost for attribute and by variable acceptance sampling (B system)

Source: own elaboration.

V. CONCLUSION

On basis of conducted simulation analyses it is not possible to interchangeably neither acceptance, nor reject stood hypothesis. Relation between total quality control costs make by acceptance sampling plans, which are presented botch analyzed ways of control depend of established distribution of weight of costs and of size of batch. When the differences between particular components of total quality control costs are not large (A systems) and additionally the size of

batch is not large, the acceptance sampling by variable is more effective. In other cases application of attribute acceptance sampling is cheaper.

Moreover in case of batch consists of mixture of elements with value of analyzed feature, which comes from different distributions, acceptance sampling plan by variable achieves better results than in case when all elements of batch are characteristic of value of analyzed feature, which comes from normal distribution with parameters agreeable with fixed norms. It is necessary to emphasize, that those case is closer to reality.

What is more acceptance sampling plans by variable even though they not always makes lower total quality control costs, than theirs attribute equivalents, they have many advantages, which may affect to improvement of quality of products. And the producer should make decision about choice of way of quality control for all products.

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WPŁYW WYBORU METODY SPRAWDZANIA NA CAŁKOWITE KOSZTY KONTROLI JAKOŚCI

Kontrola odbiorcza jest powszechnie stosowana w przedsiębiorstwach produkcyjnych, gdyż zmniejsza ona prawdopodobieństwo przekazania klientowi wadliwej partii produktów. Z wyróżnionych dwóch sposobów kontroli odbiorczej częściej wykorzystywana w praktyce jest kontrola według oceny alternatywnej. Celem niniejszego artykułu jest podkreślenie korzyści wynikających ze stosowania liczbowych planów kontroli odbiorczej.