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*Aleksandra Baszczyńska*, Dorota Pekasiewicz***

# ON SOME CONFIDENCE INTERVALS FOR POPULATION MEAN IN CASE OF ASYMMETRIC DISTRIBUTIONS OF RANDOM VARIABLES

**ABSTRACT.** In the paper we present some methods of interval estimation of the population mean of skewed population. We consider nonparametric estimation method where information about the value of asymmetry coefficient is used. We apply simulation methods to compare the lengths of confidence intervals obtained by the considered method and the classical one.

**Key words:** confidence interval, asymmetric distribution, asymmetry coefficient.

## I. INTRODUCTION

Classical confidence intervals for population mean $\mu$ of random variable $X$ is constructed on the basis of $n$ independent and identically distributed univariate sample $X_1, X_2, ..., X_n$, using distribution of statistic $t = \frac{\bar{X} - \mu}{S}\sqrt{n}$, where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2}$. When random variable $X$ has normal distribution then statistic $t$ has $t$-Student distribution with ($n$-1) degrees of freedom. When distribution of random variable $X$ is unknown, but we have a large sample, then distribution of $t$ statistic is standard normal distribution. A large sample is defined differently by different authors – in the paper the sample of minimum 30 elements is treated as large.

Skewness of distribution of random variable $X$, may cause asymmetry of distribution of random variable $t$. In this case, asymmetry coefficient of variable has opposite sign against asymmetry coefficient of variable $X$. Therefore arithmetic

* Ph. D., Chair of Statistical Methods, University of Łódź.
** Ph. D., Chair of Statistical Methods, University of Łódź.

mean from sample should not be in the middle of confidence interval. When population is positively skewed, arithmetic mean should be closer to the left side of the confidence interval, but in the case of negatively skewed population – arithmetic mean should be closer to the right side. Thus, for random variable of strong asymmetry, even in case of a large sample, we should use estimation methods which take into consideration asymmetry of distribution of the analyzed random variable. There are some methods that fulfill the above, including modifications of the classical confidence interval for the population mean. This method relies on shifting the sides of the confidence interval about the value using information about real or estimated asymmetry coefficient (cp.: Johnson N., (1978); Rousson V., Choi E., (2003); Baszczyńska A., Pekasiewicz D.,(2007)).

## II. MODIFICATION OF THE CLASSICAL CONFIDENCE INTERVAL FOR THE POPULATION MEAN

Let $X$ be random variable with skewed continuous distribution and $\mu$ population mean. On the basis of sequence $x_1,...,x_n$ values of non-complex sample $X_1,...,X_n$, for given confidence coefficient $1-\alpha$, we consider confidence interval for the population mean $\mu$.

For large sample sizes and small degrees of asymmetry, the application of classical estimation means estimation of the population mean with coverage approximate $1-\alpha$. Large magnitude of the asymmetry of the random variable $X$ may cause that we will have confidence interval for the population mean $\mu$, but the value of confidence coefficient will not be equal to $1-\alpha$. It means that with multiple repetition of this procedure, percentage of intervals that do not include real value of population mean will be bigger than $\alpha \cdot 100\%$. The modification of classical estimation method causes that confidence coefficient will be on a given level $1-\alpha$.

One of the methods that can be applied for interval estimation for population mean $\mu$ is based on Edgeworth expansion for statistic $t$ of the following form (cp. Rousson V., Choi E.(2003)):

$$P(t \le x) = \Phi(x) + n^{-\frac{1}{2}}\gamma\left(ax^2 + b\right)\phi(x) + O\left(n^{-1}\right), \tag{1}$$

where: $a = \frac{1}{3}$, $b = \frac{1}{6}$, $\gamma$ is asymmetry coefficient defined as the third central moment divided by the cube of the standard deviation, $\Phi$ and $\phi$ are the cumulative distribution function and density function of the standard normal distribution, respectively.

Hall's equality (Rousson V., Choi E. (2003)):

$$P\left\{t \le x - n^{-\frac{1}{2}}\gamma\left(2x^2+1\right)/6\right\} = \Phi(x) + O\left(n^{-1}\right) \tag{2}$$

leads to an explicit formula for confidence interval for population mean $\mu$:

$$P\left\{\overline{x} - \left(u_{1-\alpha/2} - \frac{\hat{\gamma}\left(2u^2_{1-\alpha/2}+1\right)}{6\sqrt{n}}\right)\frac{s}{\sqrt{n}} \le \mu \le \overline{x} - \left(u_{\alpha/2} - \frac{\hat{\gamma}\left(2u^2_{\alpha/2}+1\right)}{6\sqrt{n}}\right)\frac{s}{\sqrt{n}}\right\} = 1-\alpha \tag{3}$$

where:

$\overline{x}$, $s$ are arithmetic mean and standard deviation calculated from the $n$-size sample,

$u_{\alpha/2}$, $u_{1-\alpha/2}$ are percentiles of standard normal distribution of order $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ respectively, $\hat{\gamma} = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{s^3}$ is estimated, on the basis of $n$-element sample, asymmetry coefficient.

The sides of confidence interval (3) are shifted, in relation to sides of classical confidence interval, of a value $\Delta = \frac{\hat{\gamma}\left(2u^2_{1-\alpha/2}+1\right)s}{6n}$.

For $\hat{\gamma} \approx 0$ one can obtain classical confidence interval:

$$P\left\{\overline{x} - \frac{u_{1-\alpha/2}s}{\sqrt{n}} \le \mu \le \overline{x} - \frac{u_{\alpha/2}s}{\sqrt{n}}\right\} = 1-\alpha.$$

The length of confidence interval (3) is $L = 2\frac{u_{1-\alpha/2}s}{\sqrt{n}}$. It is the same value as the length of classical confidence interval for population mean.

## III. THE SIMULATION ANALYSIS OF MODIFIED INTERVAL ESTIMATION FOR POPULATION MEAN – THE DESCRIPTION OF THE EXPERIMENT

To make the analysis of modified method of estimation of population mean and to compare it with classical method, two groups of experiments were carried out.

In the first group of experiments, the populations were generated using gamma distribution with density function:

$$f(x)=\begin{cases}\dfrac{1}{\lambda^{p}\Gamma(p)}x^{p-1}\exp\left(-\dfrac{x}{\lambda}\right) & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}.$$

The following values of parameters were used: $\lambda=0,5\cdot k$, where $k\in\{1,...,6\}$ and $p=0,2\cdot s$ for $s\in\{1,...,10\}\cup\{30,...,40\}$. In particular, for $p$=1 population has exponential distribution, and for $\lambda=2$ and $p=1,2,6,7,8$ has $\chi^2$ distribution with $2p$ degrees of freedom. The values of parameters of gamma distribution were chosen in such a way, that in the simulation study different skewness of population is analyzed. In that way, J-shaped asymmetry as well as small asymmetry can be considered in that group of experiments, but all of them are characterized by positive asymmetry.

In the second group of experiments the population was generated using beta distribution with density function:

$$f(x)=\begin{cases}\dfrac{1}{B(p,q)}x^{p-1}(1-x)^{q-1} & \text{for } 0\le x\le 1\\ 0 & \text{for } x<0 \vee x>1\end{cases}.$$

The parameters of this distribution were fixed in order to use populations, in contrary to group 1 of experiments, of negative asymmetry. For beta distribution the following values of parameters are used: $p\in\{2,...,6\}$, $q=0,2\cdot s$, where $s\in\{1,...,20\}\cup\{30,...,40\}$.

From the population described above, large samples were taken and for fixed confidence coefficients, confidence intervals for population mean were computed using the methods mentioned earlier. Every estimation procedure was repeated 10000 times. In the experiment, arithmetic means of the lengths of obtained intervals and shifted values $\Delta$ were computed. Moreover, confidence coefficient was estimated as proportion of intervals which cover real parameter of population's distribution.

Applying the two considered estimation methods led to obtaing confidence intervals of the same length. In the case of classical interval estimation, on account of skewness of $t$ statistic, the estimation with smaller confidence was obtained than the assumed. That is why, the influence of modification of classical confidence interval on proportion of intervals containing population mean, is studied.

## IV. THE RESULTS OF THE MONTE CARLO ANALYSIS

For the considered population and for samples of 30, 50, 70 and 100 elements the results of the simulation study is presented below.

Table 1 and 3 contain parameters of population's distribution, estimated asymmetry coefficients. The means of interval's lengths are also presented.

Table 2 and 4 present comparison of classical method and modified one in the term of the value of estimated confidence coefficient for two groups of experiments.

Table 1

Estimated asymmetry coefficients and mean of interval's lengths for confidence coefficient 0,95 for group 1 of experiments for chosen samples sizes.

| Parameters of population's distribution | | | | Estimated asymmetry coefficients | | | | Means of interval's lengths | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $p$ | $E(X)$ | $\gamma$ | 30 | 50 | 70 | 100 | 30 | 50 | 70 | 100 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 0,5 | 0,2 | 0,1 | 4,472 | 2,574 | 3,026 | 3,230 | 3,502 | 0,144 | 0,117 | 0,098 | 0,085 |
| | 0,4 | 0,2 | 3,162 | 2,030 | 2,318 | 2,460 | 2,669 | 0,214 | 0,169 | 0,142 | 0,123 |
| | 0,6 | 0,3 | 2,582 | 1,731 | 1,962 | 2,114 | 2,230 | 0,269 | 0,208 | 0,178 | 0,150 |
| | 0,8 | 0,4 | 2,236 | 1,538 | 1,759 | 1,851 | 2,003 | 0,311 | 0,244 | 0,204 | 0,176 |
| | 1,0 | 0,5 | 2,000 | 1,414 | 1,571 | 1,680 | 1,706 | 0,352 | 0,271 | 0,233 | 0,193 |
| | 1,2 | 0,6 | 1,826 | 1,305 | 1,470 | 1,570 | 1,603 | 0,379 | 0,299 | 0,255 | 0,213 |
| | 1,4 | 0,7 | 1,690 | 1,203 | 1,341 | 1,449 | 1,493 | 0,416 | 0,320 | 0,273 | 0,231 |
| | 2,0 | 1,0 | 1,414 | 1,039 | 1,158 | 1,232 | 1,241 | 0,497 | 0,387 | 0,332 | 0,274 |
| | 4,0 | 2,0 | 1,000 | 0,729 | 0,834 | 0,852 | 0,910 | 0,705 | 0,550 | 0,464 | 0,396 |
| | 6,0 | 3,0 | 0,816 | 0,631 | 0,677 | 0,714 | 0,780 | 0,871 | 0,671 | 0,569 | 0,478 |
| | 15,0 | 7,5 | 0,516 | 0,384 | 0,462 | 0,457 | 0,512 | 1,361 | 1,069 | 0,910 | 0,757 |
| 1,0 | 0,2 | 0,2 | 4,472 | 2,588 | 3,001 | 3,255 | 3,485 | 0,286 | 0,228 | 0,201 | 0,170 |
| | 0,4 | 0,4 | 3,162 | 2,035 | 2,358 | 2,503 | 2,629 | 0,430 | 0,340 | 0,289 | 0,242 |
| | 0,6 | 0,6 | 2,582 | 1,717 | 2,000 | 2,083 | 2,218 | 0,527 | 0,427 | 0,350 | 0,301 |
| | 0,8 | 0,8 | 2,236 | 1,504 | 1,770 | 1,838 | 1,970 | 0,620 | 0,487 | 0,414 | 0,348 |

Table 1 (cont.)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1,0 | 1,0 | 1,0 | 2,000 | 1,421 | 1,593 | 1,675 | 1,747 | 0,694 | 0,545 | 0,457 | 0,389 |
| | 1,2 | 1,2 | 1,826 | 1,312 | 1,464 | 1,525 | 1,636 | 0,774 | 0,600 | 0,505 | 0,424 |
| | 1,4 | 1,4 | 1,690 | 1,214 | 1,380 | 1,419 | 1,501 | 0,835 | 0,652 | 0,546 | 0,454 |
| | 2,0 | 2,0 | 1,414 | 1,030 | 1,104 | 1,223 | 1,230 | 0,992 | 0,768 | 0,655 | 0,545 |
| | 4,0 | 4,0 | 1,000 | 0,737 | 0,834 | 0,877 | 0,927 | 1,397 | 1,103 | 0,933 | 0,783 |
| | 6,0 | 6,0 | 0,816 | 0,613 | 0,671 | 0,718 | 0,780 | 1,737 | 1,346 | 1,141 | 0,959 |
| | 15,0 | 15,0 | 0,516 | 0,387 | 0,443 | 0,486 | 0,480 | 2,751 | 2,134 | 1,807 | 1,505 |
| 1,5 | 0,2 | 0,3 | 4,472 | 2,588 | 3,037 | 3,234 | 3,547 | 0,437 | 0,358 | 0,298 | 0,258 |
| | 0,4 | 0,6 | 3,162 | 2,048 | 2,355 | 2,500 | 2,618 | 0,653 | 0,507 | 0,430 | 0,358 |
| | 0,6 | 0,9 | 2,582 | 1,736 | 1,972 | 2,104 | 2,216 | 0,796 | 0,636 | 0,536 | 0,452 |
| | 0,8 | 1,2 | 2,236 | 1,564 | 1,750 | 1,900 | 1,914 | 0,944 | 0,733 | 0,625 | 0,516 |
| | 1,0 | 1,5 | 2,000 | 1,391 | 1,596 | 1,668 | 1,716 | 1,037 | 0,817 | 0,690 | 0,579 |
| | 1,2 | 1,8 | 1,826 | 1,297 | 1,471 | 1,504 | 1,618 | 1,136 | 0,901 | 0,756 | 0,637 |
| | 1,4 | 2,1 | 1,690 | 1,220 | 1,383 | 1,447 | 1,519 | 1,240 | 0,975 | 0,823 | 0,691 |
| | 2,0 | 3,0 | 1,414 | 1,038 | 1,154 | 1,206 | 1,239 | 1,498 | 1,172 | 0,991 | 0,818 |
| | 4,0 | 6,0 | 1,000 | 0,746 | 0,830 | 0,841 | 0,927 | 2,114 | 1,653 | 1,387 | 1,175 |
| | 6,0 | 9,0 | 0,816 | 0,603 | 0,695 | 0,725 | 0,749 | 2,603 | 2,031 | 1,723 | 1,437 |
| | 15,0 | 22,5 | 0,516 | 0,405 | 0,438 | 0,456 | 0,476 | 4,113 | 3,169 | 2,701 | 2,261 |

Source: Own's calculations

Table 2

Estimated confidence coefficients for classical and modified methods for chosen sample sizes for group 1 of experiments.

| Parameters of population's distribution | | Sample sizes | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $n = 30$ | | $n = 50$ | | $n = 70$ | | $n = 100$ | |
| λ | $p$ | Modified method | Classical method | Modified method | Classical method | Modified method | Classical method | Modified method | Classical method |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 0,5 | 0,2 | 0,866 | 0,851 | 0,894 | 0,884 | 0,902 | 0,898 | 0,914 | 0,911 |
| | 0,4 | 0,897 | 0,888 | 0,918 | 0,911 | 0,922 | 0,924 | 0,930 | 0,925 |
| | 0,6 | 0,917 | 0,910 | 0,929 | 0,925 | 0,929 | 0,928 | 0,936 | 0,931 |
| | 0,8 | 0,919 | 0,910 | 0,929 | 0,927 | 0,936 | 0,933 | 0,939 | 0,939 |
| | 1,0 | 0,921 | 0,918 | 0,931 | 0,929 | 0,940 | 0,941 | 0,942 | 0,939 |
| | 1,2 | 0,924 | 0,921 | 0,934 | 0,929 | 0,940 | 0,938 | 0,940 | 0,939 |
| | 1,4 | 0,925 | 0,923 | 0,932 | 0,930 | 0,938 | 0,939 | 0,947 | 0,945 |
| | 2,0 | 0,929 | 0,924 | 0,937 | 0,935 | 0,939 | 0,939 | 0,941 | 0,942 |
| | 4,0 | 0,941 | 0,937 | 0,938 | 0,939 | 0,942 | 0,943 | 0,949 | 0,946 |
| | 6,0 | 0,941 | 0,942 | 0,943 | 0,943 | 0,940 | 0,939 | 0,944 | 0,943 |
| | 15,0 | 0,940 | 0,939 | 0,941 | 0,943 | 0,943 | 0,943 | 0,948 | 0,948 |
| 1,0 | 0,2 | 0,867 | 0,853 | 0,889 | 0,883 | 0,903 | 0,899 | 0,916 | 0,914 |
| | 0,4 | 0,903 | 0,896 | 0,912 | 0,908 | 0,920 | 0,915 | 0,930 | 0,930 |
| | 0,6 | 0,915 | 0,904 | 0,927 | 0,922 | 0,930 | 0,929 | 0,936 | 0,931 |
| | 0,8 | 0,917 | 0,915 | 0,926 | 0,924 | 0,936 | 0,935 | 0,936 | 0,936 |
| | 1,0 | 0,918 | 0,915 | 0,931 | 0,926 | 0,935 | 0,937 | 0,937 | 0,938 |

Table 2 (cont.)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 1,0 | 1,2 | 0,924 | 0,920 | 0,935 | 0,934 | 0,940 | 0,936 | 0,940 | 0,940 |
| | 1,4 | 0,925 | 0,925 | 0,934 | 0,932 | 0,942 | 0,941 | 0,943 | 0,941 |
| | 2,0 | 0,926 | 0,928 | 0,937 | 0,935 | 0,940 | 0,938 | 0,944 | 0,945 |
| | 4,0 | 0,931 | 0,933 | 0,939 | 0,936 | 0,941 | 0,942 | 0,945 | 0,947 |
| | 6,0 | 0,940 | 0,940 | 0,942 | 0,947 | 0,944 | 0,945 | 0,944 | 0,943 |
| | 15,0 | 0,944 | 0,944 | 0,944 | 0,945 | 0,946 | 0,945 | 0,945 | 0,945 |
| 1,5 | 0,2 | 0,862 | 0,854 | 0,888 | 0,878 | 0,900 | 0,895 | 0,920 | 0,913 |
| | 0,4 | 0,895 | 0,889 | 0,916 | 0,911 | 0,921 | 0,919 | 0,932 | 0,930 |
| | 0,6 | 0,908 | 0,902 | 0,921 | 0,919 | 0,926 | 0,920 | 0,944 | 0,938 |
| | 0,8 | 0,920 | 0,916 | 0,930 | 0,927 | 0,933 | 0,933 | 0,937 | 0,936 |
| | 1,0 | 0,926 | 0,918 | 0,931 | 0,930 | 0,936 | 0,935 | 0,943 | 0,940 |
| | 1,2 | 0,926 | 0,922 | 0,934 | 0,930 | 0,939 | 0,938 | 0,937 | 0,936 |
| | 1,4 | 0,930 | 0,928 | 0,936 | 0,934 | 0,940 | 0,939 | 0,940 | 0,939 |
| | 2,0 | 0,931 | 0,931 | 0,938 | 0,936 | 0,945 | 0,942 | 0,942 | 0,942 |
| | 4,0 | 0,937 | 0,937 | 0,947 | 0,947 | 0,944 | 0,943 | 0,944 | 0,946 |
| | 6,0 | 0,938 | 0,936 | 0,940 | 0,942 | 0,945 | 0,945 | 0,946 | 0,945 |
| | 15,0 | 0,938 | 0,939 | 0,942 | 0,942 | 0,945 | 0,945 | 0,945 | 0,944 |

Source: Own's calculations.

Table 3

Estimated asymmetry coefficients and mean of interval's lengths for confidence coefficient 0,95 for group 2 of experiments for chosen samples sizes

| Parameters of population's distribution | | | | Estimated asymmetry coefficients | | | | Means of interval's lengths | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $p$ | $q$ | $E(X)$ | $\gamma$ | 30 | 50 | 70 | 100 | 30 | 50 | 70 | 100 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 3,0 | 0,2 | 0,938 | –2,849 | –2,238 | –2,436 | –2,606 | –2,663 | 0,080 | 0,063 | 0,054 | 0,046 |
| | 0,4 | 0,882 | –1,844 | –1,566 | –1,690 | –1,749 | –1,775 | 0,108 | 0,084 | 0,071 | 0,061 |
| | 0,6 | 0,833 | –1,370 | –1,190 | –1,247 | –1,274 | –1,313 | 0,122 | 0,096 | 0,081 | 0,068 |
| | 0,8 | 0,789 | –1,073 | –0,917 | –0,977 | –1,021 | –1,051 | 0,131 | 0,101 | 0,087 | 0,073 |
| | 1,0 | 0,750 | –0,861 | –0,745 | –0,808 | –0,831 | –0,883 | 0,137 | 0,108 | 0,090 | 0,076 |
| | 1,2 | 0,714 | –0,698 | –0,609 | –0,633 | –0,656 | –0,667 | 0,140 | 0,109 | 0,093 | 0,078 |
| | 1,4 | 0,682 | –0,567 | –0,495 | –0,536 | –0,531 | –0,547 | 0,142 | 0,110 | 0,094 | 0,078 |
| | 2,0 | 0,600 | –0,286 | –0,254 | –0,269 | –0,264 | –0,277 | 0,143 | 0,111 | 0,094 | 0,079 |
| | 2,4 | 0,556 | –0,153 | –0,141 | –0,147 | –0,153 | –0,145 | 0,140 | 0,109 | 0,092 | 0,077 |
| | 2,8 | 0,517 | –0,046 | –0,040 | –0,037 | –0,054 | –0,045 | 0,137 | 0,106 | 0,080 | 0,075 |
| 4,0 | 0,2 | 0,952 | –3,125 | –2,310 | –2,621 | –2,718 | –2,903 | 0,064 | 0,051 | 0,042 | 0,036 |
| | 0,4 | 0,909 | –2,067 | –1,662 | –1,841 | –1,896 | –1,962 | 0,086 | 0,068 | 0,057 | 0,048 |
| | 0,6 | 0,870 | –1,574 | –1,309 | –1,421 | –1,479 | –1,489 | 0,100 | 0,078 | 0,066 | 0,056 |
| | 0,8 | 0,833 | –1,267 | –1,062 | –1,144 | –1,170 | –1,206 | 0,110 | 0,085 | 0,072 | 0,061 |
| | 1,0 | 0,800 | –1,050 | –0,892 | –0,944 | –0,992 | –1,008 | 0,116 | 0,090 | 0,076 | 0,064 |
| | 1,2 | 0,769 | –0,884 | –0,768 | –0,814 | –0,861 | –0,831 | 0,119 | 0,093 | 0,079 | 0,066 |
| | 1,4 | 0,741 | –0,751 | –0,634 | –0,695 | –0,706 | –0,726 | 0,122 | 0,096 | 0,081 | 0,068 |

Table 3 (cont.)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 4,0 | 2,0 | 0,667 | –0,468 | –0,396 | –0,448 | –0,423 | –0,447 | 0,126 | 0,098 | 0,083 | 0,070 |
| | 2,4 | 0,625 | –0,334 | –0,275 | –0,301 | –0,314 | –0,315 | 0,126 | 0,098 | 0,083 | 0,070 |
| | 2,8 | 0,588 | –0,228 | –0,197 | –0,214 | –0,214 | –0,221 | 0,126 | 0,099 | 0,082 | 0,069 |
| 5,0 | 0,2 | 0,962 | –3,320 | –2,381 | –2,689 | –2,879 | –2,940 | 0,053 | 0,042 | 0,036 | 0,029 |
| | 0,4 | 0,926 | –2,224 | –1,731 | –1,901 | –2,005 | –2,076 | 0,072 | 0,056 | 0,047 | 0,040 |
| | 0,6 | 0,893 | –1,717 | –1,385 | –1,502 | –1,570 | –1,593 | 0,085 | 0,067 | 0,057 | 0,046 |
| | 0,8 | 0,862 | –1,404 | –1,145 | –1,242 | –1,311 | –1,319 | 0,093 | 0,073 | 0,062 | 0,051 |
| | 1,0 | 0,833 | –1,183 | –1,002 | –1,077 | –1,136 | –1,136 | 0,100 | 0,078 | 0,066 | 0,055 |
| | 1,2 | 0,806 | –1,015 | –0,845 | –0,911 | –0,966 | –0,656 | 0,105 | 0,081 | 0,068 | 0,057 |
| | 1,4 | 0,781 | –0,881 | –0,750 | –0,795 | –0,839 | –0,531 | 0,108 | 0,084 | 0,071 | 0,059 |
| | 2,0 | 0,714 | –0,596 | –0,493 | –0,523 | –0,571 | –0,264 | 0,113 | 0,088 | 0,075 | 0,063 |
| | 2,4 | 0,676 | –0,463 | –0,393 | –0,405 | –0,432 | –0,153 | 0,115 | 0,089 | 0,076 | 0,063 |
| | 2,8 | 0,641 | –0,356 | –0,304 | –0,316 | –0,350 | –0,054 | 0,116 | 0,089 | 0,076 | 0,063 |

Source: Own's calculations.

Table 4

Estimated confidence coefficients for classical and modified methods for chosen sample sizes for group 2 of experiments.

| Parameters of population's distribution | | Sample sizes | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $n = 30$ | | $n = 50$ | | $n = 70$ | | $n = 100$ | |
| $p$ | $q$ | Modified method | Classical method | Modified method | Classical method | Modified method | Classical method | Modified method | Classical method |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3,0 | 0,2 | 0,904 | 0,886 | 0,921 | 0,910 | 0,928 | 0,921 | 0,938 | 0,929 |
| | 0,4 | 0,931 | 0,918 | 0,935 | 0,927 | 0,941 | 0,933 | 0,943 | 0,936 |
| | 0,6 | 0,936 | 0,926 | 0,943 | 0,936 | 0,942 | 0,937 | 0,950 | 0,942 |
| | 0,8 | 0,941 | 0,930 | 0,949 | 0,940 | 0,946 | 0,941 | 0,944 | 0,941 |
| | 1,0 | 0,935 | 0,943 | 0,951 | 0,947 | 0,951 | 0,946 | 0,951 | 0,946 |
| | 1,2 | 0,941 | 0,932 | 0,950 | 0,946 | 0,946 | 0,943 | 0,952 | 0,951 |
| | 1,4 | 0,944 | 0,935 | 0,946 | 0,943 | 0,950 | 0,946 | 0,950 | 0,947 |
| | 2,0 | 0,947 | 0,939 | 0,947 | 0,942 | 0,953 | 0,949 | 0,949 | 0,947 |
| | 2,4 | 0,947 | 0,939 | 0,947 | 0,943 | 0,953 | 0,951 | 0,948 | 0,946 |
| | 2,8 | 0,945 | 0,941 | 0,950 | 0,947 | 0,948 | 0,945 | 0,946 | 0,945 |
| 4,0 | 0,2 | 0,894 | 0,882 | 0,917 | 0,903 | 0,928 | 0,920 | 0,935 | 0,924 |
| | 0,4 | 0,924 | 0,907 | 0,937 | 0,923 | 0,941 | 0,935 | 0,940 | 0,937 |
| | 0,6 | 0,933 | 0,921 | 0,942 | 0,934 | 0,946 | 0,941 | 0,951 | 0,944 |
| | 0,8 | 0,938 | 0,929 | 0,943 | 0,934 | 0,945 | 0,940 | 0,948 | 0,946 |
| | 1,0 | 0,944 | 0,936 | 0,943 | 0,939 | 0,948 | 0,941 | 0,946 | 0,944 |
| | 1,2 | 0,942 | 0,936 | 0,944 | 0,940 | 0,949 | 0,944 | 0,950 | 0,947 |
| | 1,4 | 0,942 | 0,934 | 0,944 | 0,942 | 0,949 | 0,945 | 0,952 | 0,949 |
| | 2,0 | 0,942 | 0,935 | 0,948 | 0,943 | 0,947 | 0,944 | 0,951 | 0,948 |

Table 4 (cont.)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 4,0 | 2,4 | 0,944 | 0,940 | 0,947 | 0,944 | 0,947 | 0,945 | 0,947 | 0,945 |
| | 2,8 | 0,944 | 0,939 | 0,949 | 0,945 | 0,948 | 0,946 | 0,950 | 0,948 |
| 5,0 | 0,2 | 0,887 | 0,870 | 0,915 | 0,901 | 0,921 | 0,909 | 0,936 | 0,928 |
| | 0,4 | 0,924 | 0,907 | 0,926 | 0,922 | 0,935 | 0,932 | 0,943 | 0,937 |
| | 0,6 | 0,934 | 0,922 | 0,942 | 0,935 | 0,943 | 0,936 | 0,943 | 0,941 |
| | 0,8 | 0,933 | 0,923 | 0,942 | 0,934 | 0,948 | 0,944 | 0,946 | 0,944 |
| | 1,0 | 0,936 | 0,932 | 0,946 | 0,940 | 0,950 | 0,946 | 0,946 | 0,945 |
| | 1,2 | 0,937 | 0,933 | 0,949 | 0,943 | 0,952 | 0,948 | 0,949 | 0,946 |
| | 1,4 | 0,939 | 0,931 | 0,946 | 0,941 | 0,953 | 0,948 | 0,944 | 0,942 |
| | 2,0 | 0,945 | 0,938 | 0,946 | 0,943 | 0,946 | 0,940 | 0,951 | 0,951 |
| | 2,4 | 0,946 | 0,941 | 0,944 | 0,939 | 0,950 | 0,945 | 0,947 | 0,946 |
| | 2,8 | 0,943 | 0,939 | 0,947 | 0,943 | 0,949 | 0,947 | 0,947 | 0,946 |

Source: Own's calculations

## V. CONCLUSIONS

On the basis of the above data it is possible to conclude that the length of the regarded confidence intervals (which is the same for both methods) depends on the sample sizes. For larger samples, means of lengths of confidence intervals becomes smaller. But on the other hand, the length of intervals depends on the value of asymmetry coefficients, but only by the value of population mean. For all the analyzed sample sizes applying modification of the classical method causes that proportion of cases for which confidence interval includes the population mean becomes larger. However, this proportion, especially for J-shaped asymmetry of population, is not equal to fixed confidence coefficient. Comparing the classical method with the modified one it should be noticed that in all the considered cases of skewed population (both positive and negative), the interval using information about the real or estimated coefficient of asymmetry, assures better estimation of population mean. It applies even to population with small asymmetry.

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*Aleksandra Baszczyńska, Dorota Pekasiewicz*

## ESTYMACJA PRZEDZIAŁOWA WARTOŚCI OCZEKIWANEJ ASYMETRYCZNYCH ROZKŁADÓW ZMIENNYCH LOSOWYCH

W pracy przedstawiono pewną metodę estymacji przedziałowej średniej dla populacji o asymetrycznym rozkładzie. Rozważano metodę nieparametryczną wykorzystującą informacje o rzeczywistej lub oszacowanej wartości współczynnika asymetrii populacji. Za pomocą metod symulacyjnych dokonano porównania rozważanej metody z metodą klasyczną poprzez analizę długości przedziałów ufności oraz analizę odsetka przedziałów pokrywających szacowany parametr.