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The Application of Discriminant Analysis to the Identification of Key Factors of the Development of Polish Cities

Abstract: Due to limited resources, effective urban development policies require the identification of key development areas and priorities. The existing development strategies or results of statistical analyses can be used for this purpose. In the latter case, one of methods of multidimensional analysis can be used – discriminant analysis. Although it is applied to many areas on a microeconomic scale, e.g. in predicting the bankruptcy of enterprises, it was rarely used to assess the competitive position or the dynamics of development of cities. The main aim of the paper is to identify the most important factors of development of Polish cities with powiat status and to analyse changes of these factors in time. Apart from typical areas, such as investment, income, employment, debt, or migration, the analysis uses qualitative variables which allow us to assess whether the size of the city and its location determine the dynamics of city development. The authors have found that the key factors determining the development of the largest Polish cities are related to the situation on the labour market and investments incurred by companies as well as by the cities themselves.

Keywords: development of cities, discriminant analysis, determinants of city growth

JEL: O18, C38, R11

1. Introduction

The dominance of the service sector, especially in highly developed countries, caused by changes in consumption patterns and technologies, is the most important reason for the growing role of cities in shaping economic growth. Concentration of services in urban agglomerations is accompanied by an increase in the population living in cities, which additionally accelerates their development. As a result of these processes, we observe innovative processes mainly in cities and owe most of the generated domestic product to them (Heffner, Gibas, 2013). At the same time, we are witnessing differentiated growth of urban centres, characterised by faster development of medium and large cities due to economies of scale (Dziaduch, 2012; Sun et al., 2015). However, we can also find opinions that the growth of the population of cities is not determined by the initial number of their inhabitants, but by the characteristics of the region in which they are located (Dorocki, 2012). The growth of cities is also associated with the occurrence of many negative effects. Most often these externalities include problems related to transport, waste management and environmental pollution. The solution to these negative aspects of urbanisation in the future could be the concepts of "smart and green cities" (Wiśniewski, 2013; Kola-Bezka, Czupich, Ignasiak-Szulc, 2016).

The theory of economics indicates a relatively wide range of factors that determine the success and development of cities (see e.g. Męczyński, Konecka-Szydłowska, Gadziński, 2010; Silicon Valley Index, 2010). These include, but are not limited to, location – often measured by distance from markets and natural resources, transport costs, investment, education, the level of trade exchange, the level of social development, or technological innovation (compare the empirical analysis based on a dataset of 123 Brazilian agglomerations provided by da Mata et al., 2005 or the World Bank report related to Asia and Pacific cities, Baker, Gadgil, 2017) as well as climate change (OECD, 2010). At the same time, many authors indicate a strong correlation between social and environmental factors of urban growth and the level of economic development of cities (Jałowiecki, 2015: 159).

The detailed descriptions of areas and indicators used in the assessment of the level of urban development for the purpose of creating their rankings can be found, among others, in the works of McManus (2012) and Lopez Ruiz et al. (2014). The high degree of diversity of determinants used in urban development research is a result of the need to assess not only the level and dynamics of urban development but also their economic and ecological effectiveness (Deilmann et al., 2016). Different sets of key factors of urban development limit the comparability of city rankings (Fanni, Khakpour, Heydari, 2014; Mavrič, Tominc, Bobek, 2014), and as a result constitute a barrier to the correct interpretation of obtained rankings (McManus, Haughton, 2006).

The main aim of the paper is to identify the key factors determining the development of the largest Polish cities[1]. The research question is: could the common and stable factors of urban development be observed over the long period of time? The analysis covered 65 cities with powiat status, and the survey period covered the years 2010–2017. This quite long data period enables the assessment of the stability of the set of the most important determinants of the development of Polish cities over time. The initial set of variables concerns the following areas of cities' functioning: demography, housing, entrepreneurship, the level of inhabitants' incomes, investments made by local government and firms, location and tourism.

2. Method applied

In the analyses of the level of urban development, various research methods are used. Some of these methods are based on ratio or index analysis, such as the Silicon Valley Index (2010) or the aggregate index of Perkal (Runge, 2007), but the majority of these methods come from the area of multidimensional analyses. The most commonly used methods include: linear ordering, factor analysis, cluster analysis, correspondence analysis (Batóg, Batóg, 2017), data envelopment analysis and the analytic hierarchy process. Multidimensional discriminant analysis (MDA) was used in the study, which, according to the authors' knowledge, is a novelty in the analyses concerning the level of urban development[2]. Discriminant analysis is a quite popular method in classification of regional units based on the level of their economic development (see, for example, Jaba et al., 2006) as well as in the identification of the variables that contribute significantly to the assessment of spatial disparities in the standard of living (El-Hanjouri, Hamad, 2015). We can also find several studies where linear discriminant analysis has been used to classify urban areas. Wentz et al. (2010) have shown that discriminant analysis outperforms strict spectral classifiers in classification of urban areas images. Some authors have applied discriminant analysis to differentiate urban and non-urban land to analyse the gap between available resources and personnel and the urban expansion level in Malaysia (Elhadary, Samat, 2015). Discriminant analysis can be also used to identify the impact of differences in spatial accessibility on the development of the built environment in cities (Borzacchiello, Nijkamp, Koomen, 2010).

1 The term "development" is most often used to assess the situation of such objects as cities, regions and countries. However, there are also other terms in the literature that are used in the analysis of changes in the condition of cities, e.g. "city resilience" (Drobniak, Plac, 2015).

2 Most applications of discriminant analysis on a micro-scale are related to the prediction of companies' bankruptcy, the assessment of the financial situation of firms (Batóg, Wawrzyniak, 1997) or the identification of factors that determine the rate of return on the capital market (Batóg, Batóg, 2012).

Multiple discriminant analysis deals with multiple dependent variables – multiple groups are analysed. The aim of discriminant analysis is to examine whether a set of p variables (X_1, …, X_p) is capable of distinguishing (discriminating) among g groups. The result of multiple discriminant analysis are discriminant functions. These functions are the linear combinations of the discriminant variables on the base of which the groups are maximally distinguished (Tacq, 2007). It means that coefficients of linear combinations (β) satisfy the conditions of maximisation of the ratio of between group variance (B) to within group variance (W) (McLachlan, 2004; Panek, 2009):

$$\hat{\beta} = W^{-1}B . \tag{1}$$

The formula for canonical discriminant functions is given by Eq. 2.

$$D_{kj} = \beta_{0j} + \beta_{1j}x_{1k} + \ldots + \beta_{pj}x_{pk} , \tag{2}$$

where:
D_{kj} – value of j-th canonical discriminant function for the observation k,
x_{ik} – value of i-th discriminant variable for the observation k,
$k = 1, …, n$, n – number of observations,
$j = 1, …, r$, r – number of discriminant functions,
$i = 1, …, p$, p – number of discriminant variables,
β_{ij} – parameters of canonical discriminant function.

The number of discriminant function (r) is maximally equal to min(g – 1, p).

In order to find parameters of canonical discriminant function, canonical correlation analysis is applied. The problem is limited to solving the system of equations:

$$\left(W^{-1}B - \lambda I\right)\hat{\beta} = 0 , \tag{3}$$

where λ is called an eigenvalue, by using the characteristic equation:

$$det\left(W^{-1}B - \lambda I\right), \tag{4}$$

to calculate a maximum value for λ and find the respective vector $\hat{\beta}$.

The successive functions and canonical roots are determined (the term root refers to the eigenvalues that are associated with the respective canonical function). It is possible to test significance for roots by means of χ^2 test. In the first step, the test for all roots is conducted. Then the highest root is removed and the significance of the remaining roots is tested. These procedure is continued to the last root.

The quality of obtained discriminant function is examined by means of Wilks' Lambda statistic. This statistic is computed for the whole model and also for the models without a given variable and with only one variable (partial). The value 0 of Wilks' Lambda means perfect discrimination and the value 1 means no discrimination. Partial Wilks' Lambda is a measure associated with the unique contribution of the respective variable to the discriminatory power of the model. The F statistic is used for testing significance of the whole model and also for deciding whether given variable should be incorporated into the model (F to remove).

The additional measure of the importance of a given discriminant variable is *Tolerance*. *Tolerance* is computed as $1 - R^2$ where R^2 is the square of correlation coefficient of the given variable with all other variables in the model.

For every group, the classification function is computed also as a linear combination of discriminant variables (Johnson, Wichern, 2007). The given observation is classified into a known (*a priori*) group for which the value of classification function is the highest. The accuracy of discrimination could be evaluated on the base of *Count* R^2 given by the formula:

$$Count\ R^2 = \frac{n_0}{n} \cdot 100\%, \tag{5}$$

where:
n – number of observations,
n_0 – number of properly classified observations.

Count R^2 is interpreted as the share of properly classified observations in the total number of observations[3].

3. Data and results

The variables used in the analyses characterise the social, economic and demographic situation of the examined cities in the years 2010–2017. This shorter period in relation to the existing data in the Local Data Bank provided by the Statistics Poland results from the lack of comparability of data from previous years due to the correction of the population, including city residents, within the census carried out in 2011.

The studied cities with powiat status were divided into classes within three different variants, using two grouping variables: total revenue of the city *per capita* (Y_1) and own revenue of the city *per capita* (Y_2). In the first two variants, four

3 We can find several modifications of standard discriminant analysis that enable us to improve the quality of classification. Some of them are based on transformations of a priori or a posteriori probabilities, while some are related to direct incorporation of spatial relations in discriminant function (Batóg, 2009).

groups of objects were distinguished based on quartiles of examined variables. In the third variant (Y), the groups were received as follows:

1) group 11 – cities with total revenue *per capita* less or equal to the median and own revenue *per capita* less than or equal to the median;
2) group 12 – cities with total revenue *per capita* less than or equal to the median and own revenue *per capita* greater than the median;
3) group 21 – cities with total revenue *per capita* greater than the median and own revenue *per capita* less than or equal to the median;
4) group 22 – cities with total revenue *per capita* greater than the median and own revenue *per capita* greater than the median.

The following potential independent variables were considered:

X_1 – dependency ratio (the non-working age population per 100 persons of working age),
X_2 – new buildings completed per 1000 population,
X_3 – entities of the national economy by the REGON register per 10000 population,
X_4 – average monthly gross wages and salaries in enterprises,
X_5 – registered unemployment rate,
X_6 – employed persons per 1000 population,
X_7 – investment property expenditure of local government *per capita*,
X_8 – investment outlays in enterprises *per capita*,
X_9 – ascertained crimes by the police in completed preparatory proceedings per 1000 population,
X_{10} – distance from the capital of voivodship,
X_{11} – occupancy rate of bed places.

The obtained results of conducted discriminant analysis are presented in Tables 1–6 and Figures 1–3. Table 1 presents variables contributing significantly to discrimination of cities, basic measures of the quality of discrimination and accuracy of classifications for all analysed years for the variant in which the role of the grouping variable was played by the total revenue of the city *per capita*.

There is a relatively high level of proper classification of cities into particular groups, oscillating between 63–74%. The key factors of city development in this variant include the following variables: X_6, X_7, X_5 and X_8. They concern the situation on the labour market and the size of investments. Table 2 presents detailed results describing the discriminant strength of the variables with the highest discriminant capacity in 2014, and Figure 1 presents the canonical values for the first two discriminant functions in 2014.

The results of the analysis carried out in the second variant, when the grouping variable was own revenue of the city *per capita*, are presented in a similar layout as in the first variant in Tables 3–4 and in Figure 2. One can note slightly higher quality of the model and a higher level of proper classification of objects.

The most important determinants of urban development in this variant were the following variables: X_5, X_8, X_7 and X_4. In comparison to the first variant, the variable X_6, which describes the level of employment, was replaced by the variable X_4, which describes the level of workers' wages and salaries.

Table 1. Results of the discriminant analysis in 2010–2017 grouping variable – total revenue of the city *per capita*

Year	Influential variables	Wilks' Lambda	χ^2 (p = 0.000)	Count R^2 (%)
2010	X_1, X_6, X_7	0.41	51.84	56.92
2011	X_5, X_6, X_7, X_8	0.33	64.98	64.62
2012	X_6, X_7	0.27	76.85	64.62
2013	X_2, X_4, X_5, X_6, X_7, X_8	0.20	92.02	73.85
2014	X_4, X_5, X_7, X_8	0.23	84.15	69.23
2015	X_5, X_6, X_7, X_8	0.25	79.33	72.31
2016	X_5, X_6	0.28	73.65	69.23
2017	X_6	0.32	66.77	63.08

Source: own calculations

Table 2. Results of discriminant analysis for dependent variable Y_1 in 2014 (Wilks' Lambda = 0.234, F(24.157) = 4.252, p = 0.000)

Variable	Wilks' Lambda	Partial Wilks' Lambda	F to remove	p	Tolerance	1 – Tolerance
X_4	0.273	0.857	3.004	0.038	0.587	0.413
X_5	0.271	0.863	2.851	0.046	0.432	0.568
X_7	0.367	0.638	10.216	0.000	0.792	0.208
X_8	0.274	0.856	3.034	0.037	0.681	0.319

Source: own calculations

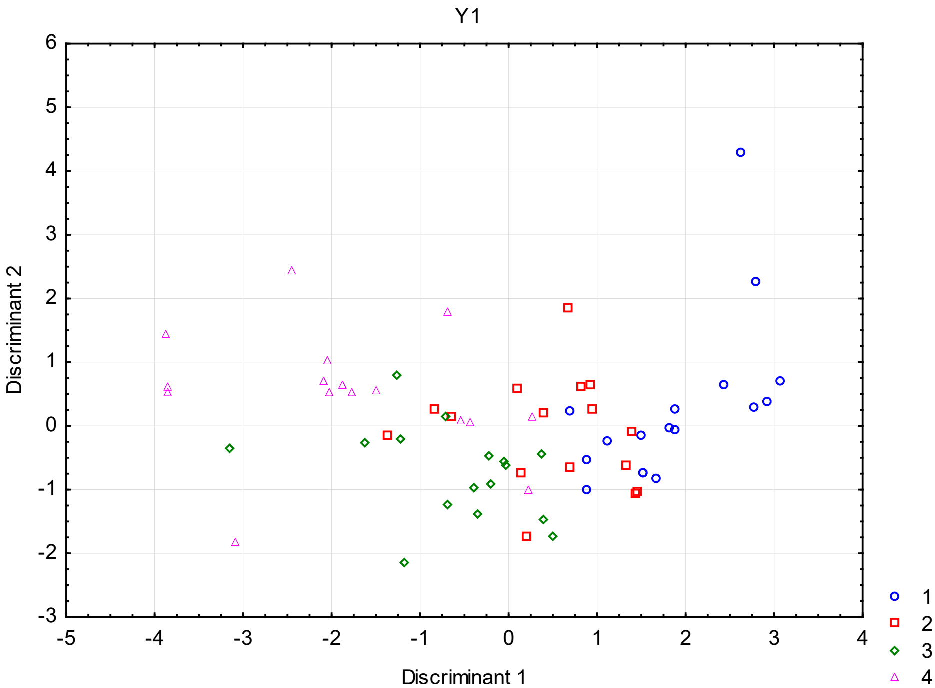

Figure 1. Cities in discriminant space for dependent variable Y_1 in 2014 (4 groups)

Source: own calculations

Table 3. Results of the discriminant analysis in 2010–2017 grouping variable – own revenue of the city *per capita*

Year	Influential variables	Wilks' Lambda	$\chi^2(p = 0.000)$	Count R^2 (%)
2010	X_5, X_8	0.29	72.05	69.23
2011	X_5, X_8	0.23	85.47	69.23
2012	X_5	0.32	65.51	66.15
2013	X_1, X_5, X_8	0.23	84.93	70.77
2014	X_1, X_7, X_8	0.23	84.03	66.15
2015	X_3, X_4, X_5, X_7, X_8	0.18	101.02	75.38
2016	X_4, X_5	0.17	103.22	69.23
2017	X_4, X_5, X_7	0.20	95.59	63.08

Source: own calculations

Table 4. Results of discriminant analysis for dependent variable Y_2 in 2014 (Wilks' Lambda = 0.235, F(24.157) = 4.245, p = 0.000)

Variable	Wilks' Lambda	Partial Wilks' Lambda	F to remove	p	Tolerance	1 – Tolerance
X_1	0.276	0.851	3.163	0.032	0.589	0.411
X_7	0.274	0.859	2.964	0.040	0.785	0.215
X_8	0.294	0.799	4.515	0.007	0.796	0.204

Source: own calculations

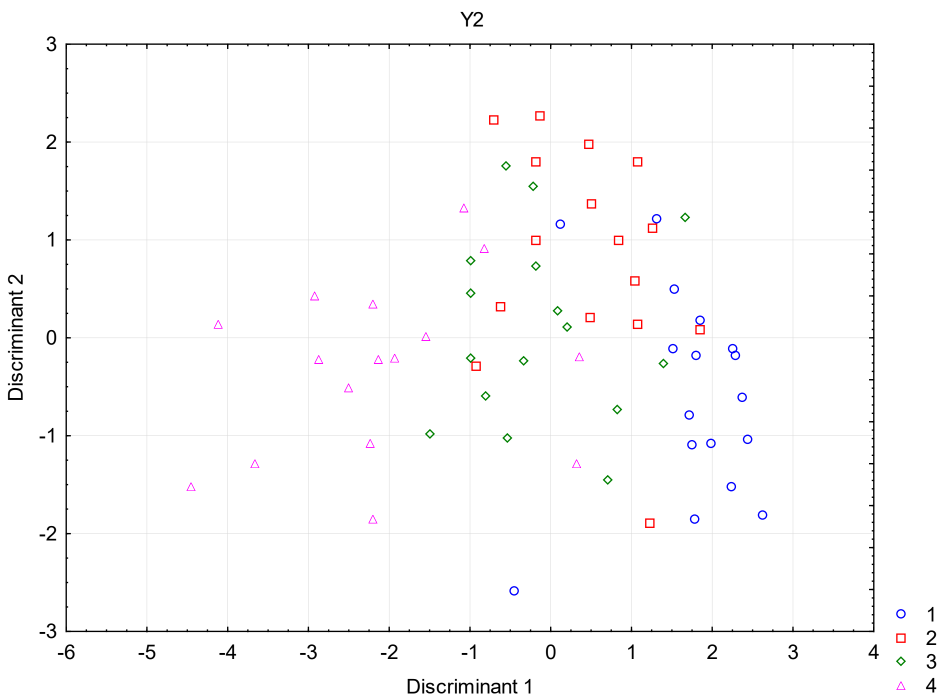

Figure 2. Cities in discriminant space for dependent variable Y_2 in 2014 (4 groups)
Source: own calculations

The same layout of results as in the previous two variants is presented for the third one in Tables 5–6 and in Figure 3. The results obtained show the best quality of the classification among all variants. The most important discriminating variables in this case are: X_5, X_6 and X_7.

Table 5. Results of the discriminant analysis in 2010–2017 grouping variable – combination of total revenue of the city *per capita* and own revenue of the city *per capita*

Year	Influential variables	Wilks' Lambda	$\chi^2(p = 0.000)$	Count R^2 (%)
2010	X_5	0.37	56.97	69.23
2011	X_5, X_7	0.28	73.38	73.85
2012	X_6, X_7	0.32	65.95	72.31
2013	$X_2, X_4, X_5, X_6, X_7, X_8$	0.23	84.82	78.46
2014	X_2, X_5, X_7, X_8	0.25	80.28	73.85
2015	X_5, X_6, X_7	0.28	73.31	76.92
2016	X_6	0.33	63.55	73.85
2017	X_4, X_6	0.35	62.08	66.15

Source: own calculations

Table 6. Results of discriminant analysis for dependent variable *Y* in 2014 (Wilks' Lambda = 0.251, F(24.157) = 4.007, p = 0.000)

Variable	Wilks' Lambda	Partial Wilks' Lambda	F to remove	p	Tolerance	1 – Tolerance
X_2	0.298	0.840	3.429	0.023	0.534	0.466
X_5	0.305	0.821	3.912	0.013	0.446	0.554
X_7	0.335	0.747	6.092	0.001	0.823	0.177
X_8	0.304	0.824	3.836	0.015	0.770	0.230

Source: own calculations

The canonical values obtained allow us to conclude on the significant contribution of the first discriminatory function in distinguishing group 11 (cities with the lowest level of development) from group 22 (cities with the highest level of development), while the second function discriminates mainly against cities in groups 12 and 21.

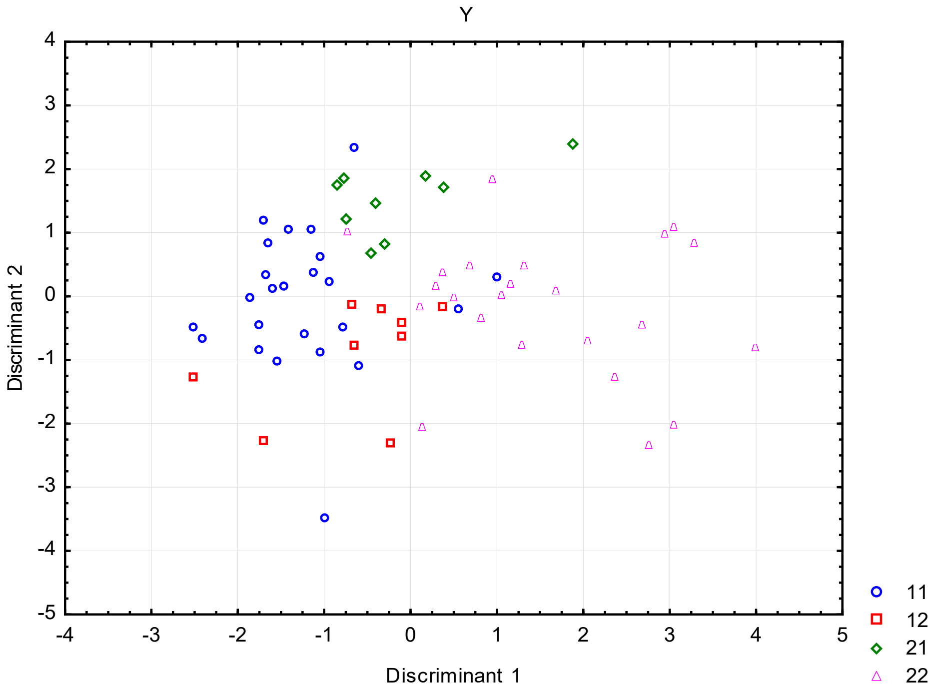

Figure 3. Cities in discriminant space for dependent variable *Y* in 2014 (4 groups)
Source: own calculations

4. Conclusions

The proposed research approach allowed us to identify very important factors determining the development of the largest Polish cities. They encompass two areas: the situation in the labour market and investment outlays incurred by companies and the cities themselves. Therefore, it can be concluded that the results obtained are consistent with the theory of regional and urban development, but also with the results of other research carried out in this area. At the same time, they provide clear information for decision-makers in charge of urban governance as to what kind of actions contribute most to the development of their cities. In one of the variants of grouping of the analysed objects, an additional factor appeared – the level of workers' income. Almost all obtained discriminant models were characterised by a satisfactory level of quality and accuracy of classification.

However, it is worth noting quite strong diversification of the sets of variables contributing the most to the discrimination of the examined cities in the analysed period. We observe that the number of variables in particular years varies from one to six and they do not repeat in every year in each variant of the classification.

Among the key factors of urban development, there were sometimes indicated variables describing the demographic structure of the population, newly created housing resources and the number of business entities. In none of the analysed years, the level of urban development was influenced by: the level of crime, the location of the city in relation to the most important urban centre in the voivodship and tourism intensity. As part of further studies, other research methods may be considered in relation to the same data. This would make it possible to compare the sensitivity of the results obtained by alternative tools. It could also be valuable to carry out an identical analysis for the period of economic crisis and assess whether the set of key determinants of urban development will change. It could be also worth considering factors influencing city development related to current climate changes.

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Zastosowanie analizy dyskryminacyjnej w identyfikacji kluczowych czynników rozwoju miast w Polsce

Streszczenie: Prowadzenie skutecznej polityki rozwoju miast przy dysponowaniu ograniczonymi środkami wymaga ustalenia najważniejszych obszarów i priorytetów rozwoju. W tym celu można wykorzystać istniejące strategie rozwoju lub oprzeć się na wynikach analiz statystycznych. W tym drugim przypadku można użyć jednej z metod analizy wielowymiarowej – analizy dyskryminacyjnej. Ma ona wiele zastosowań w skali mikroekonomicznej, m.in. w przewidywaniu bankructwa przedsiębiorstw, jednak bardzo rzadko bywa wykorzystywana do oceny pozycji konkurencyjnej lub dynamiki rozwoju miast. Głównym celem badania jest identyfikacja najważniejszych czynników rozwoju polskich miast na prawach powiatu i analiza zmian tych czynników w czasie. Oprócz typowych zmiennych, takich jak inwestycje, dochody, zatrudnienie, zadłużenie czy migracje, w analizie wykorzystane zostały zmienne o charakterze jakościowym, które pozwolą ocenić, czy wielkość miasta oraz jego lokalizacja decydują o dynamice jego rozwoju. Uzyskane wyniki wskazują, że najważniejsze determinanty rozwoju największych polskich miast związane są z sytuacją na rynku pracy i ponoszonymi nakładami inwestycyjnymi, zarówno przez firmy, jak i same miasta.

Słowa kluczowe: rozwój miast, analiza dyskryminacyjna, determinanty rozwoju miast

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