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A Stochastic Model of Investment Incentives through Tax Policy in a Transition Economy

1. Introduction

Tendency towards decline in investments has been characteristic for Russian economy since 90th. For example, volume of annual Russian investment market is 20 mld. US dollars. According to Russian Federation State Committee on Statistics, during 1992-1997 only 21.8 mld. dollars had been invested into non-financial sectors in Russia. Though investment dynamics in Russia became positive recently, total volume of investments decreased in 1997 by 5% (for comparison, in 1996 the rate of decline was 18.1%), and in the first quarter of 1998 in comparison with the same period of 1997 - by 7%. The dynamics of the direct foreign investments in Russia characterizes the increase in the credibility to Russia from foreign investors, however their absolute value is still very insignificant (less than 1% of GNP). During 1997 foreign investments into the real sector of Russian economy amounted only to 26.4 dollars per capita (at 1996 it was 14.2 doll.), i.e. dozens of times less than in Central European countries [1].

The reasons for unfavorable investment climate in Russia are well known. They have to do first of all with unstable political and economic situation (changes in the legal system are difficult to forecast, actions of federal executive bodies and subjects of federation are not synchronized), too heavy tax burden, criminal situation (including enterprise sphere). Russian and foreign investors are afraid that their projects in Russia will not be effective because:

- laws that contradict to the previous agreements can be adopted (for example elimination of tax credits);
- decisions on submission of property rights to the investor can be declared illegal;
- economic policy of the country can change as a result of change in political situation;
- after investments are made, new limitations for the entrepreneur can appear which will make him reject further use of the enterprise.

Main methods of creating incentives for the investment activity are also well known. Among them are:

- development of the system of state guarantees to the investors (for example through creation of guarantee and mortgage funds, which accumulate financial resources of the state, firms and individuals);
- development of the investment insurance system;
- tax benefits to the investors.

First two items have to do mainly with regulation of investment on the state level and now they are in Russia in the preliminary stage of development (see, for example, [2]. Use of stimulating tax policy is widely used in Russia and has already given positive results in some regions.

Though lately in mass media and legal bodies there started a campaign against tax benefits (which makes connections between tax benefit and corruption, in many cases it is really so), favorable tax policy is the only realistic and available instrument to attract foreign and domestic entrepreneurs. Additional arguments in favour of this position are results of the latest poll by consulting firm "Deloitte Tooche Tomatzu International" among multinationals, which invest in European countries.[3] The purpose of the poll was to reveal the degree of tax incentives influence on the investment decisions.

From the answers obtained from 100 corporations one can make a conclusion that though tax incentives are not a decisive factor during adoption of decisions on allocation of new investments (more important are political stability, stability of national currency, and quality of labor), but all other things equal, those countries which provide tax benefits and provide information on them are preferred. One of the main conclusions of the company experts is that "tax incentives are one of the most important weapons for incentives to foreign investors".

Among tax incentives the most important from the respondents point of view are low income taxes. They are valued much higher than accelerated amortization and export exemptions. It should be noted that income tax is a significant portion of the total tax burden and it is one of the most important revenue portions of the budget; its share in total tax revenues is more than 1/4.[4]

According to data of Russian Federation State Tax Service in 1995 income tax revenues were the largest in 10 out of 11 Russian territories (only in West-Siberian region they were slightly less than revenues from value added tax). Though lately there appeared a tendency to decrease of the income tax share, revenues from this tax in the federal budget increased by 13% during the first quarter of 1998 in comparison with the same period of the previous year (at the same time revenues from the value added tax decreased by 21%, and from excise duties - by 22%). [22]. Current Russian income tax system is characterized by federal and regional (territorial) tax rates as well as different tax exemptions. Federal tax rate is 13 %, and regional is determined by local authorities, but it should not be greater than 22% . In some regions local tax rate is lower than the upper limit, for example in St.Petersburg (20%), Tatarstan (19%), Nizhni Novgorod region (21%), Perm region (17.5%).

Among the exemptions which decrease effective tax rate the most popular are tax holidays, i.e. full or partial remission from income tax within certain interval after investments. These exemptions were used in South-Eastern Asia, Eastern Europe, some Western European countries. One of the most successful was tax holiday program in Puerto Rico, which was initiated in 1949 [6]. In the survey [4] it is pointed out that tax holidays for 10-15 years are introduced for so-called pioneer corporations specialized in high-tech production in Singapore. In Italy, starting from 1986 there have been tax holidays for the firms that are created in Southern Italy, where this exemption is a regional incentive.

In Russia tax holidays are used in more than two dozens of regions (including Novgorod region, Chuvashia, Tatarstan, Tver' region, Samara, Kaliningrad, Yekatherininburg, etc.). Mainly it was influenced by the growth of economic and political independence of the subjects of federation, which created new possibilities in order to attract investors (among them foreign) for concrete projects with the help of regional laws on tax and other exemptions, creation of local guarantee funds, easier bureaucracy procedures.

Unlike the early 90s, during last two years, foreign investors began to pay attention on the regional investment projects.

The most vivid example is Novgorod region, which created a real investment boom [7]. In December 1994 the regional Duma adopted a law “On tax exemptions to the enterprises located on the territory of the Novgorod region”. According to this law enterprises with foreign investments, which are engaged in production and are registered on the territory of the region are exempted from all regional taxes during payback period. The law gives the regional enterprises exemptions on the average up to 30% from total taxes to the local budget. Starting with the January 1, 1997 four districts of the region have declared tax-free zones, and federal part of the income tax is returned to the enterprises from the local budget. As a result in the region more than 160 enterprises with foreign investments are registered (mostly from Germany, Finland, United States). Western-Russian Regional Venture Fund of the European Bank of Reconstruction and Development gave 30 mln. dollars on investment projects financing, and 20 mln. dollars were subsidized by Italian government. As a result during three years the volume of foreign investments increased from 3.5 mln. dollars in 1994 to 154 mln. dollars in 1996. Russian companies invested in the economy of the region 40 mld. rubles. According to the evaluation of the World Bank expert, Novgorod region is among six most attractive Russian regions for foreign investments.

In economic literature there are many publications on tax incentives of already existing firms. To be precise, they deal with investigation of influence of the tax rates and other incentives, such as accelerated depreciation, tax allowances on reinvestments into the production, and tax credits. When the capital is invested abroad and foreign subsidiaries of multinationals are created, there appears an additional problem with tax credits and deferrals (depending on relation between home and host tax systems) with the purpose to avoid double taxation [11,12].

Unlike the above mentioned topics, this paper deals with tax incentives for investments into new firms. In order to clarify the motivation of our research let us explain its contents in short.

Let us consider an investment project, that assumes creation of a new enterprise in a certain region that produces certain goods and consumes certain resources. We will limit our investigation to the case when investments are direct and irreversible, i.e. they can not be taken back and used for other purposes. This investment project can be imagined as a certain consequence of expenditures and outputs in physical units (technological description of the project) in the time.

Considering prices of input and output production, the investor can calculate expected profit before he actually makes investments (virtual profit). When calculating net present value (NPV) the investor should consider various factors of risk and uncertainty based on dynamics of the virtual profit. First of all, the prices and demand on production can fluctuate stochastically depending on the situation on the market ("market, or economic" risk).

The investor behavior depends on his evaluation of the probability of the loss of the firm (and also investments) as a result of a change in the political course and creation of unacceptable situation. This factor is called "political risk".

Behavior of the investor is presumed to be rational in the sense that while looking at the virtual profit from the given project (and evaluating situation in a certain region) he can either adopt the decision on investment or postpone it for some time in order to receive information on situation in economic environment (for example on change in virtual profit). So, the objective of the investor is to choose the optimal moment for investment depending on the information that had been obtained by him before this moment.

The region can actively influence the investor behavior accelerating his appearance with the help of tax exemptions, for example appropriate interval of tax holidays. We will consider that the purpose of the region from a given project is maximization of the discounted tax payments into the regional budget in the process of the enterprise existence.

Within the proposed framework of interaction between the region and the investor the following tasks are investigated:

- determination of the optimal investment rule as a function of all parameters of the problem;
- investigation of the dependence of main economic indicators of the region, investor and federal center on the parameters of uncertainty, risk, and tax exemptions (comparative statics);
- comparison of the "optimal" (within the proposed model) tax exemptions with those existing in reality in different regions of Russia.

The starting point for this research is the McDonald-Siegel model. It is also described in detail in [14]. These papers deal with the model of investor behavior in which the profit after investment in a certain project is described by a stochastic process (geometric Brownian motion), and investments are considered to be irreversible. The purpose is to find the optimal moment for investment. Theory influence of the non-linear tax limitations on the investor behavior under uncertainty in prices and output production was studied .

Approach to this problem proposed in the above mentioned papers is related with Contingent Claims Analysis (CCA). Investment is interpreted as the purchase of the American call option on the right to make investments in the future. The expiration date of this option is the optimal moment of investment. One of the main assumptions of such model concerns the availability at the securities market of a financial asset, which price is completely correlated with the market price of the realized investment project. It is assumed that the financial market is in equilibrium, in particular it satisfies the conditions of the known Capital Assets Pricing Model (CAPM).

The model of the investor proposed in this paper is an extension of the McDonald-Siegel model by the inclusion of existing Russian income tax system, as well as political and institutional risks. However, the approach to the investigation of this problem, related to the use of the CCA methods and CAPM is incorrect for economics with undeveloped financial markets (including Russian economy). For this reason we use other methods, based on the optimal stopping theory for stochastic processes, for the investigation of the investor model. In this case optimal (according to the NPV criterion) rule is interpreted as a moment of optimal stopping for the process of virtual profit that is observed. Thus, lack of investments (investment waiting) is the consequence of the assumptions on rational behavior of the investor.

For the solution of the optimal stopping problem (for finding optimal investment level) instead of the traditional heuristic “smooth pasting” method [13,14], we propose rigorous approach based on the direct evaluation and further variation of the optimized functional. The investment rule, obtained in the analytical form, allows the region to compare different principles of determination of tax holidays, in particular to optimize their duration from the point of view of the tax payments into the regional budget depending on the parameters of the tax system.

2. The Basic Model - Investor's Problem

It will be considered a project of creation of a new enterprise (in the area of production) in a certain region of a country as an object of investment (we will say about a creation of a new enterprise, not about reconstruction of an old one, because we mean that a new taxpayer will appear). Investments (I), necessary for the project, are considered to be instantaneous and irreversible, so that they can not be taken out of the project and be used for other purposes after the project was started (sunk costs). One can think about an investment project as of a certain sequence of costs and outputs in units (technological description of the project). For this reason while looking at the current prices and output, the investor can evaluate the profit from the project; it would be (real) profit after the investment was made, and before the moment of investment one can call it (virtual) profit, i.e. hypothetical (expert) profit under the condition that the investment were made at the initial (zero) moment. By initial period of time we mean the moment when the project becomes available for investment.

The most important feature of the model is the assumption that at any moment the investor can either accept the project and start with the investment, or delay the decision before obtaining new information on the environment (prices on the production, demand, etc.), i.e. on changes in virtual profit from the project. For example, if someone wishes to invest in creation of a plant for fuel production, prices of which increase, it makes sense to delay the investment in order to receive greater virtual profit (but not so long because of the time discount effects).

Economic environment can be influenced by different stochastic factors (uncertainty in market prices, demand, etc.). For this reason we consider that the virtual profit from the project (including all taxes and payments except income tax; in this work we define profit of the firm as its taxable income) can be described by stochastic process

$\Pi = (\pi_t, 0 <= t < \infty)$

As for the lifetime of the project (duration of the existence of the new firm), it is considered infinite, however under unstable socio-political environment the investor is afraid that the project can cease to bring any profit after some time after investment, as a result of change in political or economic course of the country and the situation will be such that investor will have to refuse from further use of the enterprise. This factor is naturally called political risk, and in our model it is considered, so the investor will receive the revenues only within certain period of time after investment was made, and duration of this interval L (the investor revenue lifetime) will be a random variable.

Tax system influences the investor behavior significantly. We restrict our investigation by income taxes that bring about one fourth of all tax revenues to the state budget. According to the current Russian laws it is characterized by state (federal) and territorial (regional) tax rates as well as a set of tax benefits. In this paper we will focus on tax holidays, so the tax system can be represented as a triplet γ_f, γ_r, ν where

γ_f and γ_r are federal and regional (respectively) income tax rates,

and ν is a duration of tax holidays.

Suppose that investment of the project is started at the moment τ.

Present value of the investor return from the project can be written by the following formula

$$V_\tau = E\left(\int_\tau^{\tau+\min(\nu,L)} (1-\gamma_f)\pi_t e^{-\rho(t-\tau)}dt + \int_\tau^{\tau+\max(\nu,L)} (1-\gamma_f-\gamma_r)\pi_t e^{-\rho(t-\tau)}dt \middle| F_\tau \right) \qquad (1)$$

where ρ is a discount factor, L is the lifetime of the firm revenue for the investor, and notation E (-/F_τ) stands for the conditional expectation provided the information on the system until the moment τ.

The purpose of the investor is to find a moment for investment (rule of investing), which depends on the previous (but not the future) observations of the environment, so that its net present value (NPV) will be maximal within given tax system, i.e.

$$E(V\tau - I)e^{-\rho t} \to \max_\tau \quad (2)$$

where E=E(- | F_0) is the sign of an expectation (provided known data about the system at the moment t=0), and maximum is considered over all "investment rules", i.e. τ, depending only on observations of the environment (among them on the virtual profit from the project) until this moment (Markovian moments).

At the same time we can calculate the tax payments into the budget that can give the project after investment. Tax revenues depend on the behavior of the firm after period L of the investor income revenue ends.

One of the possible assumptions (that we are going to use further) is that though after the interval of the length L happens, say, nationalization, the firm itself continues to work and pays taxes. In this case, expected tax payments from the firm into the federal budget discounted to the moment τ are equal

$$T_\tau^f = E\left(\int_\tau^\infty \gamma_f \pi_t e^{-\eta(t-\tau)} dt | F_\tau\right) \quad (3)$$

and into the regional budget –

$$T_\tau^r = E\left(\int_{\tau+\nu}^\infty \gamma_f \pi_t e^{-\eta(t-\tau)} dt \mid F_\tau\right) \quad (4)$$

where η is the budget discount (that compares values of the budget revenues in time), that can, in general, be different from the discount ρ.

Main assumptions

As it was noted above, investor revenues lifetime L is naturally considered as a random variable. We will consider that it does not depend on the flow of the project revenues and has exponential distribution with the parameter δ, i.e. has a density $p(L)=\delta e^{-\delta L}$ Parameter δ can be interpreted as a rate of a "political risk", since it characterizes the probability of a "catastrophe" (there are no investor revenues) within a small interval of time under the condition that it never occurred in past, i.e. $P\{t<L<t+dt \mid L>t\}=\delta dt$. Note that if $\delta=0$ then investor revenues lifetime becomes infinite (there is no political risk).

The amount of investment I is fixed. Such an assumption does not restrict general considerations, and, for example, the case of exponential growth of investment (in time) can be reduced to the "constant case" with the simple shift of parameters. In [13] it was considered even more general case when I is evolved according to geometric Brownian motion, but it does not lead to a totally new pattern, it only makes formulas more complicated.

Investor's revenue from the project is described by stochastic process

$\Pi=(\pi_t,t>0)$. In order to specify it let us define $R(t,\Delta t)=\dfrac{\pi_{t+\Delta t}-\pi_t}{\pi_t}$ - the rate of growth of the project revenues at the interval $(t,t+\Delta t)$.

We will consider the process Π, which satisfies the following assumptions:

- $R(t,\Delta t)$ does not depend on F_t – "the past" of the system until moment t; (1)
- distribution of $R(t,\Delta t)$ does not depend on the moment t; (2)
- almost all trajectories π_t are positive and continuous in t. (3)

Conditions, described here, reflect some "extreme" properties of the environment, in which the project exists. For example, the first means that revenues rates of growth can not be predicted for sure on the base of "the past", and second - that they are regular stochastically. The third is the most restrictive and select profitable projects, which provide positive revenues immediately after investing (as a more weak condition one can consider a case "with a lag", when a project begins to give positive profits after some period of time after investment). It turned out that the above mentioned conditions determine stochastic revenue process Π by the unique way.

Proposition 1.

If conditions (1)-(3) are satisfied, then the process $(\pi_t, t >=0)$ is a process of geometric Brownian motion, i.e. it satisfies the following stochastic differential equation:

$$d\pi_t = \pi_t(\alpha dt + \sigma dw_t) \quad (5)$$

or, equivalently,

$$\pi_t = \pi_0 \exp\left\{(\alpha - \frac{\sigma^2}{2})t + \sigma w_t\right\} \tag{6}$$

where π_0 is given initial state of the process, α and σ are real numbers ($\sigma>=0$, and w_t is a standard Wiener process (Brownian motion) (for proof see [25]).

Parameters of the geometric Brownian motion α and σ have a natural economic interpretation, namely,

α is an expected instantaneous rate of revenues growth;

σ^2 is an instantaneous variance of rate of revenues growth (volatility of the project, rate of the "uncertainty").

Let us point out that the income growth rate does not have to be positive. Negative α means that the profit flow decreases with time (on the average), nevertheless it remains positive; and when $\alpha=0$, it changes around its mean π_0.

Let us point out that the process of geometric Brownian motion was introduced for the first time, probably, by P.Samuelson [17], who called it "an economic Brownian motion" and considered it as the most appropriate to describe evolution of prices in the economy. Hypothesis on geometric Brownian motion is also the basis of the modern description of the securities prices in the financial markets. In particular it lies in the foundations of well known Black-Scholes option pricing theory [18].

Now we can write the explicit formulas for investor's Present Value (1) and tax payments into the budgets. Using the known formula $Ee^{h\xi} = e^{h^2 d\xi/2}$ for the Gaussian random variable ξ with mean zero and variance $D\xi$, the relation (6) for geometric Brownian motion implies

$$E(\pi_t \mid F_\tau) = \pi_\tau e^{\alpha(t-\tau)}, \quad | \, t \geq \tau \tag{7}$$

Applying Fubini Theorem we have

$$V_t = \int_0^{\nu} \int_{\tau}^{\tau+L} (1-\gamma_f) E(\pi_t / F_\tau) e^{-\rho(t-\tau)} p(L) dt dL +$$

$$+ \int_{\nu}^{\infty} (\int_{\tau}^{\tau+\nu} (1-\gamma_f) E(\pi_t \mid F_\tau) e^{-\rho(t-\tau)} dt +$$

$$+ \int_{\tau+\nu}^{\tau+L} (1-\gamma_f-\gamma_r) E(\pi_t \mid F_\tau) e^{-\rho(t-\tau)} dt) x P(L) dL = \pi_\tau \frac{1-\hat{\gamma}(\nu)}{\mu-\alpha} \tag{8}$$

where $\mu=\rho+\delta$, $\hat{\gamma}(\nu) = \gamma_f + \gamma_r e^{-(\mu-\alpha)\nu}$

Two last lines in these relations show that μ=ρ+σ can be viewed as political risk adjusted discount rate of the investor, and δ as "political risk premium" (on the problem of measuring the effect of political risk see also [19]).

Analogously we can obtain explicit formulas for the expected tax payments into federal and regional budgets (3) and (4):

$$T_\tau^r = \int_{\tau+\nu}^{\infty} \gamma_r e^{-\eta(t-\tau)} E(\pi_t|F_\tau)dt = \gamma_r \pi_\tau \int_{\tau+\nu}^{\infty} e^{-(\eta-\alpha)(t-\tau)}dt = \frac{\gamma_r \pi_\tau}{\eta-\alpha} e^{-(\eta-\alpha)\nu} \quad (9)$$

$$T_\tau^f = \int_{\tau}^{\infty} \gamma_f e^{-\eta(t-\tau)} E(\pi_t|F_\tau)dt = \gamma_f \pi_\tau \int_{\tau}^{\infty} e^{-(\eta-\alpha)(t-\tau)}dt = \frac{\gamma_f \pi_\tau}{\eta-\alpha} \quad (10)$$

Solution of the Investor Problem

The problem which the investor faces is an optimal stopping problem for the stochastic process. Though the relevant theory is well developed (see, for example, 20), there are very few problems, which have solution in the explicit form, and the problem (2) belongs to this rare type.

Let β(θ) be a positive root of the quadratic equation

$$\frac{1}{2}\sigma^2 \beta(\beta-1) + \alpha\beta - \theta = 0 \quad (11)$$

Let us point out that β (θ)>1 whenever θ>α. if σ>0 then, obviously,

$$\beta(\theta) = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\theta}{\sigma^2}}$$

If σ=0, then β(θ)=θ/α whenever a>0, and there is no positive root of the equation (11) whenever a<=0, but we find it convenient to consider β(θ)=∞. We will denote β=β(ρ), $\tilde{\beta}$=β(η)

Theorem 1.

Let the profit from the project be distributed according to the geometric Brownian motion (5), and $\rho > a$.

Then the optimal moment for the investment is: $\tau^* = \min\{t>=0: \pi_t>=\pi^*\}$ (12)

where

$\pi^*=kI(\mu-\alpha/1-\hat{\gamma}), k=\beta/\beta-1, \mu=\rho+\sigma$.

(The proof of this theorem see at [25])

Theorem 1 shows that optimal moment for the investment begins when the virtual profit achieves the critical level π^*.

Formulas of this type (for the case of geometric Brownian motion) are given in (13), (14) (for more simple model of the investor) and probably secede from the results (21), (22)(for the options).

One can look at relation (12) from investor's point of view (see formula (8)), namely, the optimal moment for the investment τ^* coincides with the moment when the investor's Present Value V_τ achieves the level kI. It means that the classical investment rule - to invest in the project, when its discounted revenues (V_τ) are greater than discounted costs (in our case, I) does not work any more for this model since the investments can be postponed, and it should be modified in the following way: "make investment when the discounted revenues are $k=\beta/\beta-1$, ($k>1$) times greater than the costs". Detailed analysis of this phenomenon, its connections with the known Jorgenson rules and Tobin's ratio q can be found in [14].

In order to avoid trivial moment of investment $\tau^*=0$, we will further suppose that initial value of the profit π_0 satisfies $\pi_0<\pi^*$.

If we know the optimal moment for the investment, we can find the optimal income of the investor as well as relevant tax payments from the project into the federal and regional budgets. Let us denote the discounted net income of the investor under the condition that he behaves optimally, i.e. maximal value of the function in (2) as A, and let $\Phi^f=E(T^f_{\tau*}e^{-\eta\tau*})$ be discounted tax payments into the federal budget under the optimal behaviour of the investor, and Φ^r be the similar value for the tax payments into the regional budget.

Let us point out that the optimal moment of investment τ^* is not always finite, i.e. the project can remain non-invested. Let us define $P^*=P(\tau^*<\infty)$ the probability of investing the project (in some finite moment of time). If the project will be invested, then expected time of the investment waiting $E\tau^*$ which characterizes an investment activity (with regard to this project), i.e. time possibilities of the investor's entry into the system, is also interesting for study.

Theorem 2.

Let the profits from the project be a process of geometric Brownian motion (5), and discounts are such that min (ρ,η) > a. Then the following formulas hold:

1. $NPV = (k-1)I\left(\frac{\pi_0}{\pi^*}\right)^{\beta}$,

2. $\Phi^f = \frac{\gamma_f \pi_0}{\eta - \alpha}\left(\frac{\pi_0}{\pi^*}\right)^{\tilde{\beta}-1}$,

3. $\Phi^r = \frac{\gamma_r \pi_0}{\eta - \alpha}\left(\frac{\pi_0}{\pi^*}\right)^{\tilde{\beta}-1} e^{-(\eta-\alpha)\nu}$,

4. $P^* = 1$ if $\alpha \geq \frac{1}{2}\sigma^2$, $P^* = \left(\frac{\pi_0}{\pi^*}\right)^{1-2\alpha/\sigma^2}$ if $\alpha \leq \frac{1}{2}\sigma^2$,

5. $E\tau^* = \frac{1}{\alpha - \sigma^2/2}\log\frac{\pi^*}{\pi_0}$ if $\alpha > \frac{1}{2}\sigma^2$, $E\tau^* = \infty$ if $\alpha \leq \frac{1}{2}\sigma^2$

In [25] had been proved that for optimal tax payments from the project we have the following formula:

$$\nu^* = \frac{1}{\mu - a}\lg\left[\frac{\gamma_z}{1-\gamma_f}\left(1 + \left(\tilde{\beta} - 1\right)\frac{\mu - a}{\pi - a}\right)\right] \qquad (13)$$

when $\nu^*>0$

Such optimal tax holidays ν* are able to bring to the region maximal discounted tax payments from the project (from income tax)

Comparative Statics: Dependence on Uncertainty, Risk, and Tax Exemptions

In this Section we will point out the general type of dependence of the main economic indicators of the model on the parameters of the project and the environment. Let us emphasize the parameters connected with the uncertainty, risk, and tax exemptions (i.e. volatility of the project σ, rate of the political risk δ, and tax holidays ν). Dependence on the other parameters has less definite character and can vary depending on the composition of the input parameters.

Table 1 describes qualitative behaviour of economic indicators as functions of σ, δ, and ν. Here, arrows indicate monotonicity (in the corresponding direction), sign ∧ means presence of the maximum (transition from increase to decrease), and sign ~ means that the qualitative behaviour does not have definite character and can vary depending on the composition of the input parameters.

Table 1. Qualitative behavior of main economic indicators as functions of volatility, political risk and tax holidays

Indicators	Volatility σ	Political risk δ	Tax holidays ν
Investment level, π^*	↑	↑	↓
Probability of investment, P^*	↓	↓	↑
Exp. Time of invest. Waiting, $E\tau^*$	↑		↓
Investor NPV, A	~	↓	↑
Federal tax payments, Φ^f	~	↓	↑
Regional tax payments, Φ^r	~	↓	either ↓ or ^

As one can see from table 1, if the volatility of the project σ increases, probability of investing (if $a<1/2\sigma^2$) falls, and expected time of investment waiting (if $a>1/2\sigma^2$) grows, that it is not obvious intuitively.

Influence of tax holidays on investor and tax payments into budgets (in relative sense) decreases. When the political risk δσ increases, the investment level $\pi^*(\nu)$ also increases. Therefore, the moment of investment $\tau^*(\nu)$ increases for all samples of the profit process (unlike the previous case of change in the parameter σ, when the process itself changes). Later entry of the investor leads to the decrease of his net discounted income and to the decrease of tax payments into the budgets but at the same time the tax holidays become more effective (they increase relative gain).

Hence, relative influence of the tax holidays increases with the increase in political risk (though relative income of the investor and tax payments are decreased) and decrease in volatility of the project. Concerning the dependence on tax holidays let us note that almost all our inferences are consistent with the intuitive notions.

Payback period approach (Novgorod Scheme)

As it was mentioned in the introduction in some regions of Russia period until break-even point (payback period) is accepted as duration of tax holidays. During the realization of the investments project, that is directed to the creation of a new production process, reconstruction or modernization of already existed one, tax exemptions from regional income tax are given until the project breaks even. We will call this exemption pattern "Novgorod scheme", since it was the most clearly manifested in the Novgorod region.

Detailed instructions on the calculation of break-even points was prepared by the consulting firm "Arthur Andersen". According to the accepted definitions the period until the break-even point is determined as minimal time interval (starting from the moment when balance profit is received), during which discounted expected cash flow (or profit) becomes equal to the initial expenditures. Depending on whether the discount factor is used in such a flow (existing instructions do not give strict recommendations on this issue), we can consider discounted payback period, or non-discounted payback period.

Within the framework of our model, the following parameters correspond to the above mentioned definitions:

non-discounted payback period - the solution of the equation

$$E\int_{\tau^*}^{\tau^*+\nu_1} \pi_t \, dt = I \tag{a}$$

discounted payback period - the solution of the equation

$$E\int_{\tau^*}^{\tau^*+\nu_2} \pi \, e^{-\rho(t-\tau^*)} dt = I \tag{b}$$

Using the obtained expressions for the optimal moment of investment one can receive simple relations that make it possible to find values ν1 and ν2 .

Non-discounted payback period.

Using the formula (7) we have

$$E\int_{\tau^*}^{\tau^*+\nu_1} \pi_t \, dt = E\int_{\tau^*}^{\tau^*+\nu_1} E(\pi_t | F_{\tau^*})\, dt = E\pi_{\tau^*} \int_{\tau^*}^{\tau^*+\nu_1} e^{\alpha(t-\tau^*)} dt = E\pi_{\tau^*} \int_0^{\nu_1} e^{\alpha t} dt = \pi^*(e^{\alpha\nu_1} - 1)/\alpha$$

(when α = 0 we will define the latter expression along the continuous as)

For this reason from (a) follows that

$$\pi^*(e^{\alpha\nu_1}-1)/\alpha = I$$

and by substituting formula for π^* from the theorem 1 we obtain the relation

$$k\frac{\mu-\alpha}{\alpha}(e^{\alpha\nu_1}-1) = 1-\gamma_f-\gamma_r e^{-(\mu-\alpha)\nu_1} \quad \text{(c)}$$

where $\mu = \rho+\delta$, $k = \dfrac{\beta}{\beta-1}$

Discounted payback period.

Similarly, we can obtain

$$E\int_{\tau^*}^{\tau^*+\nu_2} \pi_\tau e^{-\rho(t-\tau^*)}dt = E\pi_{\tau^*}\int_0^{\nu_2} e^{-(\rho-\alpha)t}dt = \pi^*[1-e^{-(\rho-\alpha)\nu_2}]/(\rho-\alpha)$$

and relation

$$k\frac{\mu-\alpha}{\rho-\alpha}[1-e^{-(\rho-\alpha)\nu_2}] = 1-\gamma_f-\gamma_r e^{-(\mu-\alpha)\nu_2}. \quad \text{(d)}$$

Thus payback periods ν_1 and ν_2 can be found as the roots of transcendental equations (a) and (b).

If $\delta=0$ then equation (d) has the explicit solution

$$\nu_2 = \frac{1}{\rho-\alpha}\log\frac{k-\gamma_r}{k-1+\gamma_f}$$

In table 2 we give some numerical calculations of payback periods and compare them with the „optimal" tax holidays for different variants of average income growth rate for the projects with high (σ=0,1), moderate (σ=0,04), and low (σ=0,01) volatility. As discount we use ρ=20 %,σ=0.

Table 2. Numerical calculation of payback periods and optimal tax holidays

a	ν_1	ν_2	ν_2
(σ = 0,1)			
-3%	3.1	5.8	4.3
-2%	3.2	5.8	3.9
-1%	3.2	5.8	3.4
0%	3.2	5.7	2.7
1%	3.2	5.6	2.1
2%	3.2	5.5	1.3
3%	3.2	5.4	0.5
(σ = 0,04)			
-3%	3.4	7.2	10.5
-2%	3.5	7.4	9.7
-1%	3.6	7.4	8.6
0%	3.6	7.3	7.1
1%	3.6	7.1	5.4
2%	3.5	6.7	3.8
3%	3.5	6.4	2.4
(σ = 0,01)			
-1%	3.7	8.4	19.1
0%	3.8	8.5	13.8
1%	3.7	7.9	8.1
1.5%	3.7	7.5	6.4
2%	3.6	7.2	5.0
3%	3.6	6.7	3.0

Analysis of the obtained results make it possible to come to the following conclusions.

1) Payback period without discount shows significant robustness to the parameters of the project, and changes very insignificantly within 3-4 years. Discounted payback period is also robust, and is greater than non-discounted about two times.

2) Values ν1 and ν2 decrease both with the increase of the average income growth rate from the project (it is obvious intuitively) and decrease of volatility of the project (uncertainty).

3) As for comparison with the optimal tax holidays (from the point of view of the region), it is impossible to prefer any of the break-even period variants (with discount or without discount). When growth rate or volatility of the project is low enough, discounted break-even period is closer to the optimal, but when parameters of the the project α and σ increase, the optimal holidays decrease, and converge with the non-discounted payback period . It means that any variants of payback periods calculations (with or without discount) can be used under certain circumstances.

Conclusions and Final Remarks

1. Consideration of risk and uncertainty factors during the estimation of effectiveness of investment projects is difficult to analyse even in the stable economies, not only in Russia. At the same time, it is obvious that these factors play a very important role in investment decisions. Official methods [24] recommend to calculate the effectiveness of the project under different scenarios of the changes in economic environment. The choice of such scenarios is performed with the expert methods and often depends on subjective opinions. The proposed model can be viewed as a very aggregated description of the investor behaviour in the economic environment, which is subjected to different stochastic fluctuations and has certain "extreme properties". Our main hypothesis is the assumption on the distribution of the investment project's profits (geometric Brownian motion), which reflects the element of unpredictability (chaotic character) of small changes in income along with their exponential growth or fall. Such a process is characterised only by two parameters, which have clear economic sense: expected instantaneous rate of income growth and its variance (volatility, rate of uncertainty).

These parameters can be evaluated on the basis of known methods of statistics and regression analysis applied for observed virtual profits.

The most restrictive in our hypothesis is the requirement for non-negative revenues after the investment, but in the first place it can be relaxed, for example with the help of introduction of lag period between the moment of investment and profit producing, and in the second place, now there are many projects (for example, in energy sector or in revival of the "frozen" technological lines) which will bring profit immediately (or within short period of time) after the investment.

2. In the proposed model of the investor's behaviour it was possible to obtain explicit solution (in the analytical form) of the investor problem, namely, to find the investment rule that maximizes investor's net present income from the

project. Using obtained formulas one can make both quantitative calculations, as well as theoretical analysis of the dependence of the above mentioned indicators on the parameters of the project and environment.

3. We compared the proposed "optimal" principle of tax holidays determination with real existing in Russian regions - "fixed tax holidays" (usually for 3-5 years) and "payback period" principles. It was shown that such a real tax holidays are enough good (in duration and effectiveness in comparison with optimal tax holidays) only for the investment projects with either rather high volatility or not very small expected income growth rate. Moreover, there is no the only "real" principle which would be good for all groups of projects. For investment projects with low expected income growth rate and not enough high volatility an optimal tax holidays are significantly longer than existing (in reality), and they can increase considerably tax payments into the regional budget.

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