ACTA UNIVERSITATIS LODZIENSIS
FOLIA OECONOMICA 48, 1985

Czesława Jackiewicz*, Halina Klepacz**, Elżbieta Żółtowska*

THE TWO-STAGE ITERATIVE METHOD FOR ESTIMATING THE CES PRODUCTION FUNCTION

1. Introduction

Consider two-factor CES production function in the form

$$f(K,L) = \alpha\left[\delta K^{-\varrho} + (1-\delta)L^{-\varrho}\right]^{-\frac{\nu}{\varrho}}, \tag{1}$$

K, L denoting outlays of capital and labour, respectively. Assuming that the output of production process Y is a random variable depending on a random term ε the two simplest stochastic models are

$$Y = f(K,L) + \varepsilon \tag{2}$$

and

$$Y = f(K,L)\, e^{\varepsilon}. \tag{3}$$

The choice of (2) or (3) for the model describing production process determines the methods of estimation that can be applied to the estimation of production function parameters.

Out of well known and commonly accepted methods of estimation, the Gauss-Newton's and Marquardt's methods are applied to the model (2) and Kmenta's to the model (3).

The methods are based on a linearization of the CES func-

*Dr., Lecturer at the Institute of Econometrics and Statistics, University of Łódź.
**M.A., Senior Assistant at the Institute of Econometrics and Statistics, University of Łódź.

tion. In Gauss-Newton's method the CES function is expanded (as a function of variables $\alpha, \delta, \nu, \rho$) in Taylor series around the point $(\alpha_0, \delta_0, \nu_0, \rho_0)$ up to the terms containing first partial derivates. Next, accepting a criterion of minimizing the sum of residual squares[1], iterative procedure is applied to find estimates a, b, c, r of paramaters $\alpha, \delta, \nu, \rho$ respectively. Gauss-Newton's method is thus a simple iterative procedure which does not ensure convergence. Marquardt's method is a combination of linearization principle with gradient method. The method implies the convergence of residual squares obtained for a linearized form.

The Kmenta's method is a one-step-procedure consisting in an expansion of the natural logarithm of the function (1) (treated as a function of ρ) in Maclaurin series with the accuracy up to the first two terms of this expansion. Investigations carried out in [3], [4] on properties of the CES function parameter estimates obtained when the above method is applied, proved the existence of the bias in parameter estimates for small samples. Apart from that the estimates showed also high variation coefficients. The worst results of estimation were obtained for the parameter ρ.

Applying Kmenta's method enables, however, getting a correct estimate of the parameter ν. Its bias and variation are small. Such a situation makes it possible to use the estimate of parameter ν as an a priori information in another CES function parameters estimation.

The method of estimation of the CES production function presented in this paper makes use of an a priori information about the value of parameter ν. The method belongs to a group of least squares methods. More precisely, the problem of seeking the minimum of a non-linear criterion-function (which is not a square form) consists in solving a non-linear system of normal equations.

---

[1] In such a procedure the sum of residual squres is a square form.

2. Estimation of CES Function Parameters by Two-Stage Iterative Method

CES production function constitutes a non-linear model for the sample $\{(Y_i, K_i, L_i), \; i = 1, \dots, n\}$. It has the form

$$(4) \qquad Y_i = \alpha\left[\delta K_i^{-\varrho} + (1-\delta)L_i^{-\varrho}\right]^{-\frac{\nu}{\varrho}} + \varepsilon_i,$$

where:

- Y - output,
- K - fixed assets,
- L - employment,
- α - parameter of the scale of production,
- δ - distribution parameter,
- ν - homogeneity parameter,
- ϱ - substitution parameter,
- ε - random term.

Taking into consideration the necessity of economic interpretation the parameters of the above function should fulfil the following conditions: $\alpha \in (0, +\infty), \delta \in (0, 1), \nu \in (0, +\infty), \varrho \in (-1, 0) \cup \cup (0, +\infty)$. For $\varrho = 0$ production function (4) becomes the Cobb--Douglas type.

The problem of estimating the parameter of function (4) using the least squares method, given a priori ν, reduces to seeking the values A, b, r such that the function

$$(5) \qquad \mathfrak{z}(A,b,r) = \sum_{i=1}^{n}\left\{Y_i - A\left[bK_i^{-r} + (1-b)L_i^{-r}\right]^{-\frac{\varrho}{r}}\right\}^2$$

attains minimum.

The existence of optimum values A, b, r of function (5) is, first of all, conditioned by the existence of the following system of equations' solution:

$$
(6)\quad
\begin{cases}
\dfrac{\partial \mathfrak{F}}{\partial A} = -2 \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i = 0, \\[2ex]
\dfrac{\partial \mathfrak{F}}{\partial b} = \dfrac{2A\nu}{r} \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i \dfrac{K_i^{-r} - L_i^{-r}}{g_i} = 0, \\[2ex]
\dfrac{\partial \mathfrak{F}}{\partial r} = -2A \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i' = 0,
\end{cases}
$$

where:

$g_i = b\, K_i^{-r} + (1 - b)L_i^{-r}$,

$G_i = (g_i)^{-\frac{\nu}{r}}$,

$G_i' = G_i \left[\dfrac{\nu \cdot \ln g_i}{r^2} - \dfrac{\nu}{r} \cdot \dfrac{g_i^*}{g_i}\right]$,

$g_i^* = -b\, K_i^{-r} \ln K_i - (1-b) L_i^{-r} \ln L_i$.

The solution of system (6) is equivalent to finding of zero-points of the system of functions

$$
(7)\quad
\begin{cases}
F_1 = \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i, \\[2ex]
F_2 = \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i \dfrac{K_i^{-r} - L_i^{-r}}{g_i}, \\[2ex]
F_3 = \sum\limits_{i=1}^{n} (Y_i - AG_i)G_i',
\end{cases}
$$

where F_1, F_2, F_3 are non-linear functions of estimates A, b, r so that the analytic way of determination of zero-points estimators' values is highly complicated and practically impossible. Only the value of A is in this case easy to be established[2] (as a function of b and r). Making use of the first equation of (6), we have

$$A = A(b,r) = \frac{\sum_{i=1}^{n} Y_i G_i}{\sum_{i=1}^{n} G_i^2}. \tag{8}$$

So the system of equations (6) assumes the form

$$\begin{cases} \sum_{i=1}^{n} (Y_i - A(b,r)\, G_i) G_i \dfrac{K_i^{-r} - L_i^{-r}}{g_i} = 0, \\ \sum_{i=1}^{n} (Y_i - A(b,r) G_i) G_i' = 0. \end{cases} \tag{9}$$

In order to solve the above system we shall apply the following iterative method.

Assuming some starting values $b = b^{(o)}$, $r = r^{(o)}$ the iterative process proceeds in such a way (where k denotes the given iteration number), that

1^o for a determined $b = b^{(k)}$ in the successive iterations such $r = r^{(k+1)}$ is found that $|F_3(b^{(k)}, r^{(k+1)})| < W$, where W is an a priori implied value, for example $W = 10^{-8}$.

2^o for the previously determined $r = r^{(k+1)}$ in the successive iterations such $b = b^{(k+1)}$ is found that $|F_2(r^{(k+1)} b^{(k+1)})| < W$;

3^o the iterations described in 1^o and 2^o are repeated by turns until for k = IT, and thus $b^{(IT)}$, $r^{(IT)}$, the absolute values of $\partial\mathfrak{z}/\partial A$, $\partial\mathfrak{z}/\partial b$, $\partial\mathfrak{z}/\partial r$ become smaller than W.

The determined values $r^{(k)}$ and $b^{(k-1)}$ are obtained as a result of successive approximations using the tangent method i.e. for determined values of $b^{(k-1)}$ and $r^{(k-1)}$ we intercept the tan-

[2] The value of A obtained in this way constitutes a good estimate of parameter α; it is supported by the results of many experiments.

gent of the function $F_3(b^{(k-1)}, r^{(k-1)})$ of the variable r; the point of intersection of the tangent and Or axis is a new value of the variable r in a first step of the internal interaction - so we get $r_1^{(k-1)}$. Next a new tangent of F_3 is constructed in point $(b^{(k-1)}, r_1^{(k-1)})$. The abscissa of the point of intersection with Or axis determines a new value $r_2^{(k-1)}$, and so on. The internal iterations are continued until in a successive step ITT such a value $r_{ITT}^{(k-1)} = r^{(k)}$ is produced that $|\partial\Phi/\partial r| < W$.

The series of values $r_j^{(k-1)}$ is iteratively calculated using the recursive formula

$$r_j^{(k-1)} = r_{j-1}^{(k-1)} - \frac{F_3(b^{(k-1)}, r_{j-1}^{(k-1)})}{M_2(b^{(k-1)}, r_{j-1}^{(k-1)})},$$

for j=1, ..., ITT, where:

$$M_2 = \frac{\partial F_3}{\partial r} = \sum_{i=1}^{n} (Y_i \cdot G_i' - A_2 G_i^2 - 2AG_i G_i') \ln\left(g_i - r\frac{g_i^*}{g_i}\right) +$$

$$- r\sum_{i=1}^{n} (Y_i - AG_i) G_i \frac{g_i^{**} g_i - (g_i^*)^2}{g_i^2},$$

$$g^{**} = bK^{-r} \ln^2 K + (1-b)L^{-r} \ln^2 L,$$

$$A_2 = \frac{\partial A}{\partial r} = \frac{\sum_{i=1}^{n} (Y_i - 2AG_i) \cdot G_i'}{\sum_{i=1}^{n} G_i^2}.$$

In the same way the value $b^{(k)}$ is established, at $r^{(k)}$ determined, i.e. $b^{(k)} = b_{ITB}^{(k-1)}$, if only $|\partial\Phi/\partial b| < W$, for $b = b_{ITB}^{(k-1)}$ and $r = r^{(k)}$. Successive values $b_j^{(k-1)}$, for j=1, ..., ITB are established using the formula

$$b_j^{(k-1)} = b_{j-1}^{(k-1)} - \frac{F_2(b_{j-1}^{(k-1)}, r^{(k)})}{M_1(b_{j-1}^{(k-1)}, r^{(k)})},$$

where:

$$M_1 = \frac{\partial F_2}{\partial b} = \sum_{i=1}^{n} (Y_i S_i - A_1 G_i^2 - 2AG_i S_i) \frac{K_i^{-r} - L_i^{-r}}{g_i} +$$

$$- \sum_{i=1}^{n} (Y_i - AG_i) G_i \frac{K_i^{-r} - L_i^{-r}}{g_i},$$

$$S = \frac{\partial G}{\partial b},$$

$$A_1 = \frac{\partial A}{\partial b}.$$

Such an iterative way of finding the solution of the system (9) is the base for elaborating the estimation method for parameters A, b, r, while criterion (5) is assumed. The method is called the two-stage iterative method (TSIM).

3. Generating of Sample Space

To establish the properties of the presented CES function estimation method a Monte-Carlo experiment was used. For this the sample space $\Omega = \{(Y, K, L)\}$ fulfilling relation (1) has been constructed. The construction consisted in finding theoretical values, of the endogenous variable YT_i, i=1, ..., n for the given values, of exogenous variables $\{(K_i, L_i), i=1, ..., n\}$ and, of parameters $\alpha, \delta, \nu, \varrho$ (assumed a priori), the theoretical values of the endogenous variable $\{YT_i, i=1, ..., n\}$ according to the formula

$$YT_i = \alpha \left[\delta K_i^{-\varrho} + (1 - \delta) L_i^{-\varrho}\right]^{-\frac{\nu}{\varrho}}.$$

The calculated values YT_i were then added to the random term ε so that the values of endogenous variable Y were equal to

$$Y_i = YT_i + \varepsilon_i, \qquad i = 1, \ldots, n.$$

The values of the random variables ε_i were generated from the normal distribution $N(0, \sigma_\varepsilon)$. The parameter σ_ε was determined as

$$\sigma_\varepsilon = S(YT)\sqrt{1/R^2 - 1},$$

where $S(YT)$ denotes the standard deviation of YT and R^2 is the coefficient determining the part of variation of variable Y being explained by YT, i.e. it is equivalent to the theoretical determination coefficient for model (1).

Drawing IP times the realizations of ε we get the sample space:

$$\Omega\left(\left\{(K_i, L_i), i = 1, \ldots, n\right\}, \alpha, \delta, \nu, \rho, R^2\right) =$$

$$= \left\{\left(Y_1^{(s)}, \ldots, Y_n^{(s)}\right) : s = 1, \ldots, IP\right\}.$$

The space Ω constitutes the base for the determination of the sequence of estimates $\{A_s\}, \{b_s\}, \{r_s\}$ ($s = 1, \ldots, IP$) of parameters α, δ, ρ generating this space.

A few types of sample spaces were investigated, the source of variation being:

1° the range of correlation of the variables K and L,

2° the values of parameters α, δ, ρ,

3° the value of R^2,

4° sample size (n),

5° the quantity IP.

In the first case the variation of the sample space consisted in the choice of two different sets of values $(K, L)_1$ and $(K, L)_2$, correlation coefficients between K and L being 0.03 and 0.97, respectively. The set $(K, L)_1$ was chosen from random number tables, but the set $(K, L)_2$ corresponds (with the accuracy to the assumed scale) to the real values of fixed assets and employment in the Polish economy in the years 1958-1977.

The set of parameters was varying in relation to ρ taking ρ =

= -0.5, -0.2, 0.2, 0.5, 1.0 for $\alpha = 2.0$, $\delta = 0.4$, $\nu = 1.0$. Such a concept is due to the economic importance of the parameter ϱ as well as to the worst estimates of this parameter obtained using other methods. For R^2 values 0.99, 0.98, 0.95 and 0.90 were assumed. Investigations were made for samples of 20, 25 and 30 elements.

On the basis of sequences of estimates $\{A_s\}$, $\{b_s\}$, $\{r_s\}$ (s = = 1, ..., IP), the properties of parameter estimates are established for every type of sample space. Mean values of the estimated bias, variation coefficient, the variance about the mean and about the real value of parameter were analysed.

4. Efficiency of the Two-Stage Iterative Method for a Deterministic Model

Initially the process of the TSIM convergence was analysed for the case of deterministic models, i.e. when $R^2 = 1.0$ (then Y = YT). The analysis was expected to give answers to the following questions:

1. Is the convergence process dependent on the choice of starting points?

2. What is an average number of iterations and average time of reaching the assumed real point?

3. In what way the iterative process is influenced by the range of correlation of exogenous variables K and L?

Numerical experiments carried out on ODRA 1304 computer using # CES 4 program proved the convergence of the iterative process. The number of iterations depends on the choice of accuracy[3] (W), as well as on the choice of starting point, more precisely on r_0, and on the range of correlation of variables. The results allow us to put out several interesting conclusions.

1^o All parameter estimates are convergent to the assumed parameter values. The only exception is in the case when starting r_0 and real ϱ are the numbers with opposite signs i.e. for

---

[3] It is obvious that the smaller the value W the larger the number of iterations.

starting $r_0 \in (-1, 0)$ and real value of parameter $\varrho > 0$ (and inversely) and increase (or decrease) up to 0 or r obtained in successive iterations is observed. And, in turn, with r tending to 0 the iterative process is no longer convergent. In every iteration the values of r "travel" about 0 which is implied by numerical properties of an expression of the following form[4]:

$$A\left[b K_1^{-r} + (1 - b)L_1^{-r}\right]^{-\frac{\nu}{r}}.$$

In this case when TSIM is applied, the sign of starting value r should be changed. If the situation is still the same after the change of the sign then the hypothesis should be accepted that the relation between the output and production factors is of the Cobb-Douglas type, i.e. a function with the elasticity of substitution $\sigma = 1$. It follows that expecting the elasticity of substitution greater than [1] we should assume positive starting r_0: expecting $\sigma < 1$ we should choose negative r_0.

2^0 The convergence of the iterative process does not depend on a choice of starting values (if only the condition on the sign of parameter r is satisfied). It was observed, that the number of iterations is smaller when starting value r is greater than ϱ for $\varrho > 0$, and smaller than ϱ for $\varrho < 0$.

3^0 The estimates of parameter δ become stable much sooner than the estimates of parameter ϱ. So that for starting b equal to δ the estimate of r is equal to the real value of parameter ϱ in the first iteration IT already.

4^0 The convergence in the proposed method does not depend on the range of correlation of exogenous variables. In both cases ($\varrho_{KL} = -0.036$, $\varrho_{KL} = 0.966$) the a priori assumed real values were obtained. But the range of correlation influences significantly the number of iterations. It is obvious that the greater the correlation coefficient, the greater the number of iterations.

On the basis of the results obtained for the deterministic model it should be stated that the two stage iterative method

---

[4] For $r = 0$ the function (1) transforms into Cobb-Douglas one.

provides good results and is efficient. However, for stochastic models the bias and effectiveness of parameter estimates should be investigated. The following part of the paper is devoted to this problem.

5. Properties of TSIM for a Stochastic Model

The sample spaces and Monte-Carlo experiments were applied to establish the properties of TSIM estimates. Particular attention was paid to basic characteristics of the obtained sequences of parameter estimates $\{A_s\}$, $\{b_s\}$, $\{r_s\}$. Especially the following values have been analysed: mean values from the IP repetitions, variances and standard deviations from the sample (calculated in relation to the mean), variation coefficients for the means, the values of bias for the means, variances and standard deviations calculated in relation to the parameters. The same characteristics have been determined for the sum of squares of residuals (Q), estimates of determination coefficient R^2 from the samples and the value F determined as a sum of squares of partial derivates of the criterion function Φ.

The obtained results, being rather preliminary, are better than expected in view of a small number of undertaken experiments. The parameter estimates were supposed to be biased for two reasons: the basic sample covered 20-elements and was a small one, and, secondly, in a common widely-accepted opinion, the iterative methods are biased. In the experiments carried out for $R^2 = 0.990$ the statistically significant parameter bias has not been observed as mean parameter estimates did not differ from the real values of these parameters more than by one standard deviation for the mean. The dependence of mean parameter estimates on the number of repetitions IP is presented in Figs 1-3.

The mean value of the sum of residual squares Q and the mean R^2 showed a very similar behaviour. The characteristic obtained in IP repetitions were identical for different starting values. The experimental results for $R^2 = 0.99$, $\alpha = 2.0$, $\delta = 0.4$, $\rho = 0.2$ obtained in successive repetitions IP = 50, 100, 200 are shown in Table 1. It is worthwhile to note that the

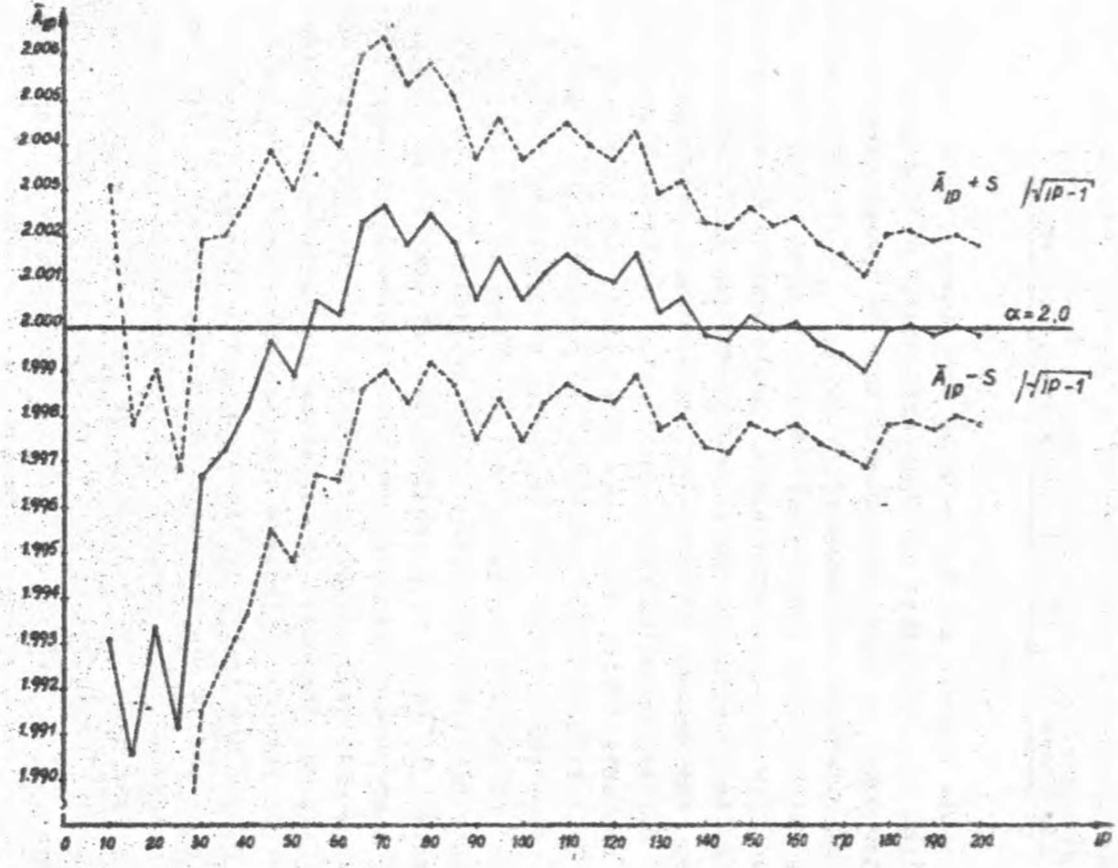

Fig. 1. Mean value of the parameters α estimates calculated after IP repetitions

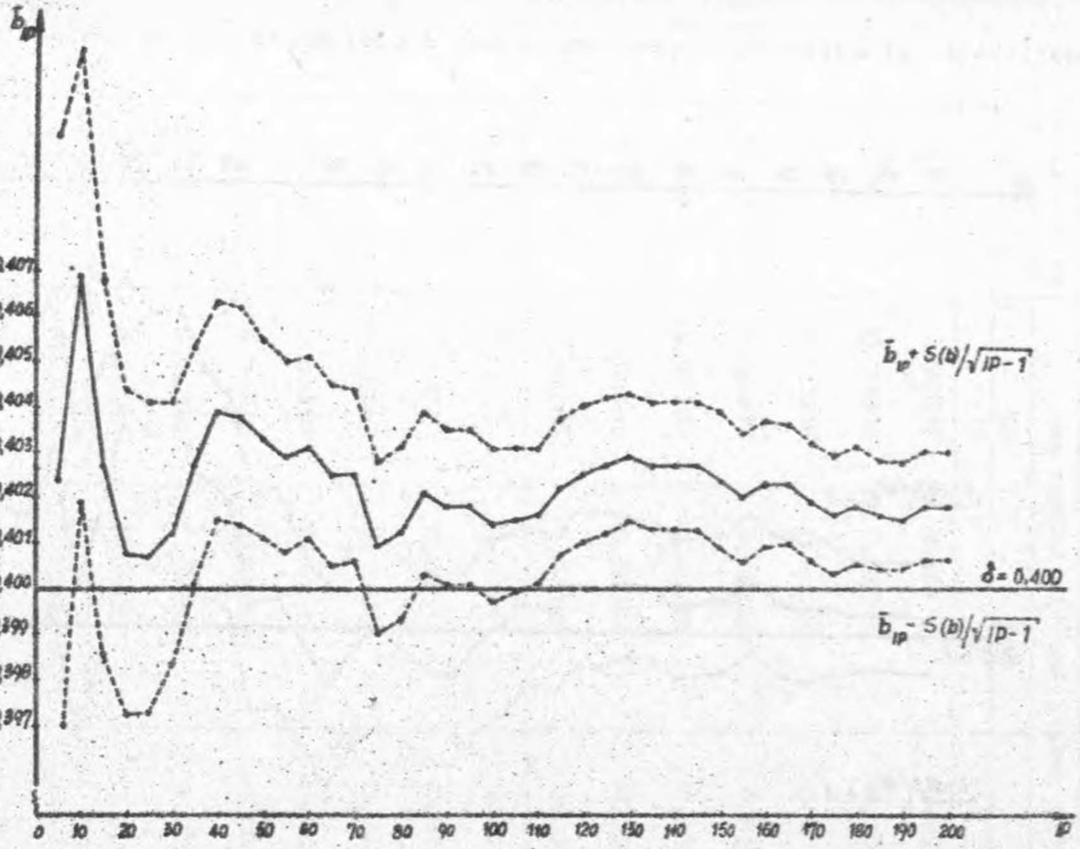

Fig. 2. Mean value of the parameters δ estimates calculated after IP repetitions

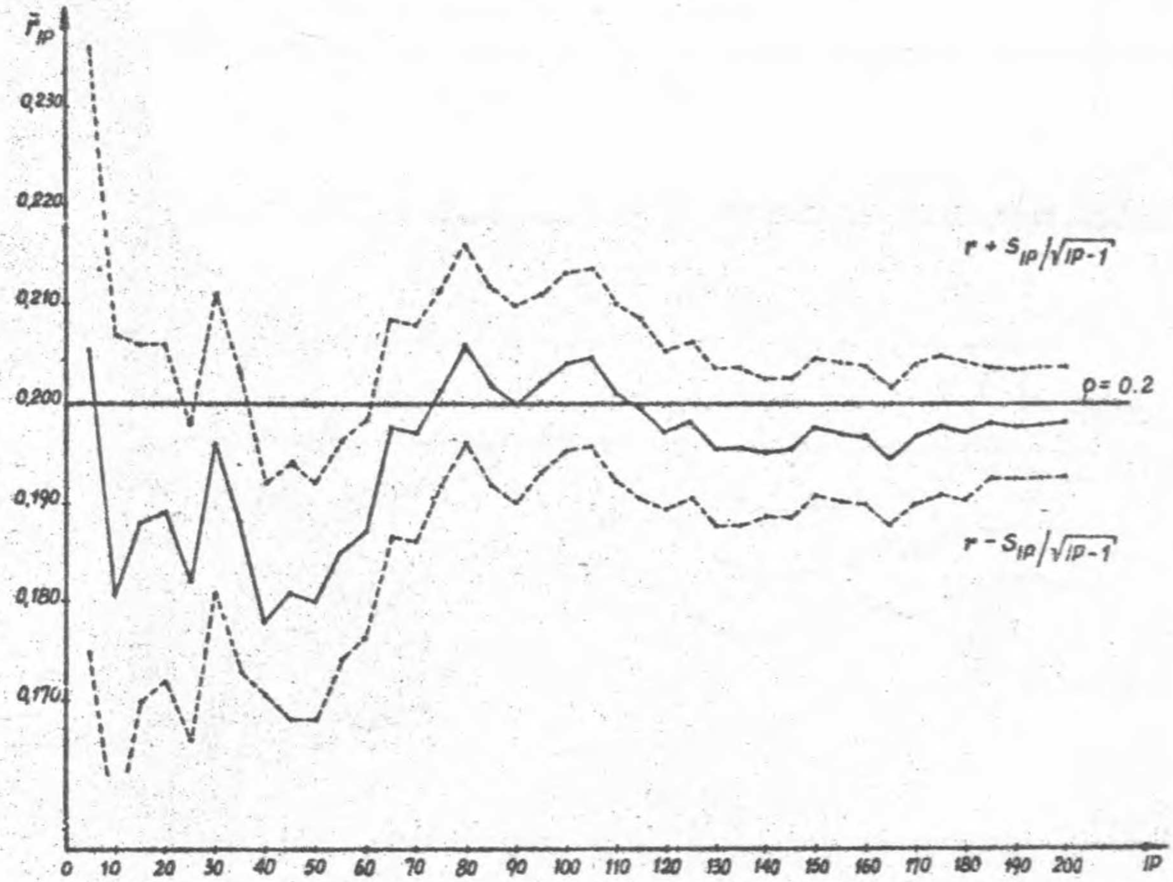

Fig. 3. Mean value of the parameters ρ estimates calculated after IP repetitions

Table 1

Mean sample parameter estimates
and their characteristics for $R^2 = 0.990$, $n = 20$

Characteristics for a sequence of estimates		The number of repetitions (IP)		
		50	100	200
1	2	3	4	5
$\{A_s\}$	$\overline{A}_{IP}$	1.99892	2.00060	1.99977
	$S^2(A)$	0.00082	0.00092	0.00076
	$S(A)$	0.02858	0.03029	0.02767
	$S(A)/\overline{A}$	0.01429	0.01513	0.01383
	$\overline{A} - \alpha$	0.00108	-0.00060	0.00023
	$S^2(A)$	0.00082	0.00092	0.00077
	$S(A)$	0.02860	0.03029	0.02767
$\{b_s\}$	$\overline{b}_{IP}$	0.40330	0.40144	0.40182
	$S^2(b)$	0.00024	0.00027	0.00028
	$S(b)$	0.01549	0.01645	0.01665
	$S(b)/\overline{b}$	0.03840	0.04097	0.04143
	$\overline{b} - \delta$	-0.00330	-0.00142	-0.00182
	$S^2(b)$	0.00025	0.00027	0.00028
	$S(b)$	0.01584	0.01651	0.01674
$\{r_s\}$	$\overline{r}_{IP}$	0.18041	0.20424	0.19790
	$S^2(r)$	0.00654	0.00855	0.00749
	$S(r)$	0.08090	0.09244	0.08656
	$S(r)/\overline{r}$	0.44843	0.45262	0.43738
	$\overline{r} - \rho$	0.01959	-0.00424	0.00210

Table 1 (contd.)

1	2	3	4	5
	$s^2(r)$	0.00693	0.00856	0.00750
	$s(r)$	0.08324	0.09254	0.08658
$\{R_s\}$	$\overline{R}^2_{IP}$	0.99150	0.99175	0.99172
	$s^2(R^2)$	$4.7 \cdot 10^{-6}$	$5.7 \cdot 10^{-6}$	$6.7 \cdot 10^{-6}$
	$s(R^2)$	0.00217	0.00238	0.00259
	$s(R^2)/\overline{R}$	0.00219	0.00240	0.00261
	$\overline{R}^2 - R^2_o$	0.00150	0.00175	0.00172
	$s^2_{R^2_o}(R^2)$	$6.9 \cdot 10^{-6}$	$8.7 \cdot 10^{-6}$	$9.6 \cdot 10^{-6}$
	$s_{R^2_o}(R^2)$	0.00264	0.00296	0.00311
$\{Q_s\}$	$\overline{Q}_{IP}$	361.67	352.22	351.41
	$s^2(Q)$	9047.37	10 803.84	11 993.04
	$s(Q)$	95.12	103.94	109.51
	$s(Q)/\overline{Q}_{IP}$	0.26	0.29	0.31
	$\overline{Q}_{IP} - Q$	-2.19	-11.64	-12.46
	$s^2_Q(Q)$	9052.17	10 939.33	12 148.20
	$s_Q(Q)$	95.14	104.59	110.22
	$\overline{F}_{IP}$	$1.58 \cdot 10^{-9}$	$1.87 \cdot 10^{-9}$	$1.82 \cdot 10^{-9}$
	$s^2(F)$	$5.21 \cdot 10^{-18}$	$5.36 \cdot 10^{-18}$	$5.82 \cdot 10^{-18}$
	$s(F)$	$2.28 \cdot 10^{-9}$	$2.31 \cdot 10^{-9}$	$2.41 \cdot 10^{-9}$
	$s(F)/\overline{F}_{IP}$	1.44	1.23	1.32
	$F_{IP} - 0$	$1.58 \cdot 10^{-9}$	$1.87 \cdot 10^{-9}$	$1.82 \cdot 10^{-9}$

Table 1 (contd.)

1	2	3	4	5
	$S_o^2(F)$	$7.73 \cdot 10^{-9}$	$8.88 \cdot 10^{-18}$	$9.13 \cdot 10^{-18}$
	$S_o(F)$	$2.78 \cdot 10^{-9}$	$2.98 \cdot 10^{-9}$	$3.02 \cdot 10^{-9}$

number of iterations in successive samplings was identical to that in the deterministic case (7 till 10). In the process of sampling there were generated such two samples (where the generator started from 0.41053835 and 0.65214471) that the number of iterations was stopped with IT = 30 and the results, especially for r, seriously differed from the real ones. The values of Q and the indicator F were large for the two cases. The two samples were not taken into account in establishing the mean estimates and their characteristics.

Conclusions concerning the characteristics of parameter estimates obtained on the basis of numerical experiments always depend on the scope of the experiments, i.e. on the number of repetitions for a basic sample. It is a priori assumed that the number IP should be indifinitely large. Practically the fulfilment of this condition is almost impossible because of time- and labour-consumption. Therefore, a limitation on the nymber of repetitions is necessary.

Interesting results follow from the analysis of empirical distributions of parameter estimates (for IP = 200). The highest stability showed the estimate of parameter α : the variation coefficient for it was equal to 1.58% (it was measured as a share of a standard deviation in the mean). The lowest stability was revealed by the estimate of parameter ρ : the variation coefficient - very high - was equal to circa 45.1%. The variation coefficient for the estimate of parameter δ was equal to 4.08%. The empirical distributions of the estimates of parameters α, δ, ρ are shown in Figs 4-6. These distributions show left-hand side asymmetry for A and right hand side asymmetry for b and r.

The above presented results constitute only a part of those which we obtained. These which are not presented here, have very similar properties. The proposed two-stage iterative method

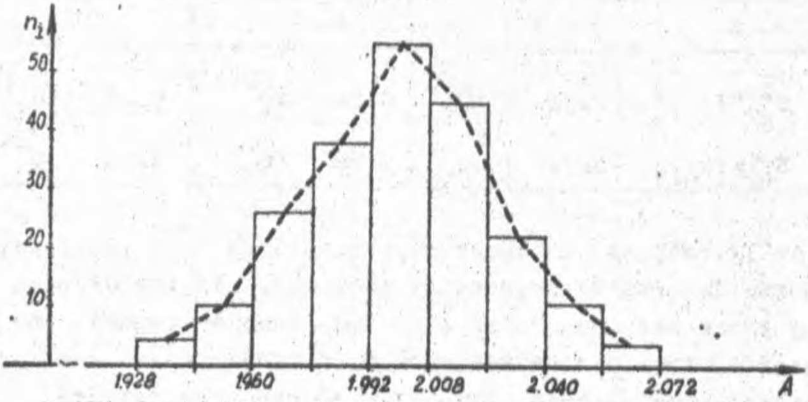

Fig. 4. Diagram of empirical distributions of estimate A for IP = 200 (R^2 = 0.990, n = 20)

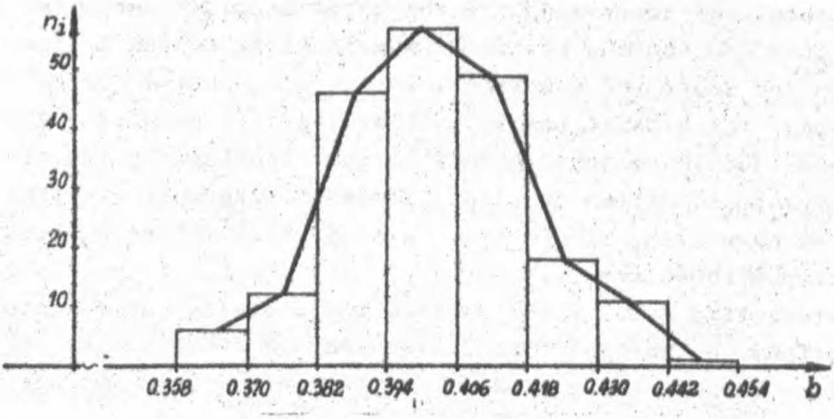

Fig. 5. Diagram of empirical distribution of estimate b for IP = 200 (R^2 = 0.990, n = 20)

can be therefore applied to estimate the parameters of CES production function, assuming that all conditions for the random term are fulfilled. The analysis of properties of this method in the case of weaker assumptions is carried out at the Institute of Econometrics and Statistics, University of Łódź.

The paper is based on the investigations carried out under the contract R.III.9.5.

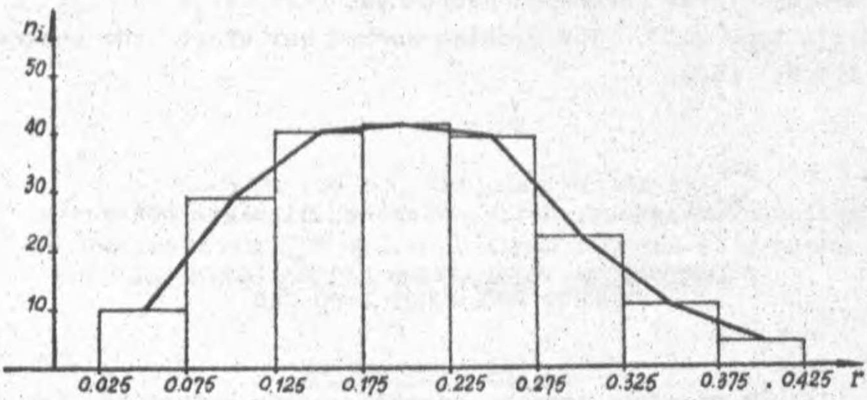

Fig. 6. Diagram of empirical distribution of estimate r for IP = 200 (R^2 = 0.990, n = 20)

BIBLIOGRAPHY

[1] J a c k i e w i c z Cz., K l e p a c z H., Ż ó ł t o w-s k a E. (1976), Estymacja funkcji produkcji typu CES przy wykorzystaniu informacji a priori uzyskanych z różnych metod estymacji tego typu funkcji, The problem worked out under the contract R.III.9, Łódź.

[2] J a c k i e w i c z Cz., K l e p a c z H., Ż ó ł t o w-s k a E. (1976), Estymacja funkcji produkcji typu CES przy wykorzystaniu informacji a priori. Metoda podwójnej i-teracji, The problem worked out under the contract R.III.9, Łódź.

[3] J u s z c z a k G., (1976), Analiza własności estymatorów parametrów funkcji produkcji typu CES otrzymanych metodą Kmenty, The problem worked out under the contract R.III.9, Łódź.

[4] K u s i a k G., Ż ó ł t o w s k a E. (1976), Estymacja nieliniowych postaci funkcji produkcji (funkcje produkcji ty-pu CES), The problem worked out under the contract R.III.9, Łódź.

[5] Ż ó ł t o w s k a E. (1976), Estymacja postaci funkcji

produkcji przy warunkach pobocznych (Estymacja funkcji produkcji typu CES), The problem worked out under the contract R.III.9, Łódź.

Czesława Jackiewicz, Halina Klepacz, Elżbieta Żółtowska

DWUSTOPNIOWA ESTYMACYJNA METODA ESTYMACJI FUNKCJI PRODUKCJI TYPU CES

W artykule przedstawiono propozycję metody estymacji funkcji produkcji CES z addytywnie wprowadzonym składnikiem losowym. Metoda ta jest oparta na klasycznej metodzie najmniejszych kwadratów. Otrzymany układ nieliniowych równań normalnych rozwiązuje się w sposób iteracyjny dwustopniowo. W artykule przedstawiono również wyniki eksperymentu Monte-Carlo uzyskane tą metodą.