MACROMODELS ‘2011

Edited by
Władysław Welfe
Aleksander Welfe

AMFET MONOGRAPHS

MODELLING ECONOMIES IN TRANSITION 2011

Edited by
Władysław Welfe
Piotr Wdowiński

Macromodels
International Conference

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MACROMODELS ‘2011

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INTRODUCTION

This monograph contains a selection of papers that were presented at the 38th International Conference *Problems of Building and Estimation of Econometric Models – MACROMODELS '2011* held jointly with the 16th AMFET Conference on Modelling Economies in Transition in Poznań, Poland between 30 November and 3 December, 2011. The organisers were: the Department (Chair) of Econometric Models and Forecasts of the Institute of Econometrics, University of Łódź, the Committee of Statistics and Econometrics, Polish Academy of Sciences, and the Association for Modelling and Forecasting Economies in Transition (AMFET).

The meetings took place in Poznań. Altogether 44 participants attended both conferences, of whom 2 were from abroad. Altogether 27 papers were delivered, 3 being invited papers, and the remaining papers were presented in 10 sessions. For publication 5 articles were accepted. The other papers were mostly published elsewhere, mainly in outstanding journals. Their available abstracts are enclosed.

Before publication the articles were reviewed. The papers accepted for publication were revised by the authors prior to the regular editorial processing.

Participants of the meetings represented in the past mainly the Central and East European and Scandinavian countries. However, there has always been a strong representation of the West European and American scholars, with the Nobel Prize Laureates – Professor Lawrence R. Klein and more recently in Cedzyna (2002) Professor Robert Engle, the Nobel Prize Laureate in 2003. In Warsaw in 2003 we hosted prominent European scholars as invited speakers, including Professors David Hendry, Stephen G. Hall, Søren Johansen, Katarina Juselius and Helmut Lütkepohl. Hence, it can be concluded that our conferences have recently become an important forum for applied econometricians from all over Europe.

The conference center helped to provide the meetings with a unique, informal atmosphere. It was to a large extent the result of the efforts of the conference organisers led by Professor Aleksander Welfe and of the Secretary of the Programme Committee – Michał Majsterek.

The conference proceedings published in this monograph present separately the articles and abstracts of Economies in Transition and MACROMODELS '2011. In order to make the perception of the contents easy the programme of the meetings is enclosed.

We would like to thank the Polish Academy of Sciences (PAN) and the Ministry of Science and Higher Education, the Faculty of Economics and Sociology and the Institute of Econometrics, University of Łódź for their financial support, and our numerous colleagues and the editorial staff for their efforts, which made it possible to publish this monograph.

Władysław Welfe

Łódź, November 2012

MACROMODELS ‘2011

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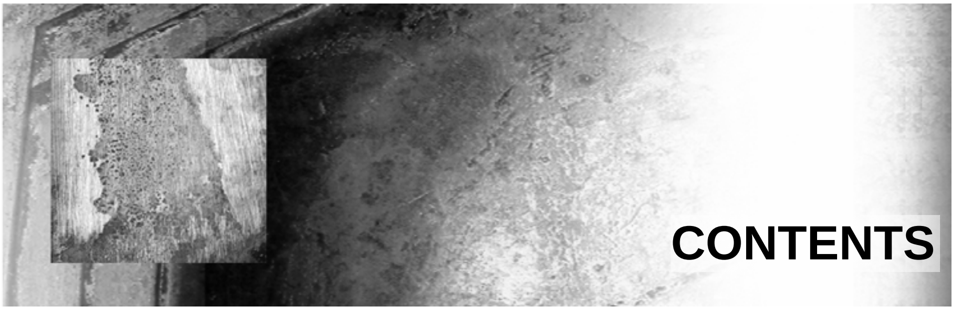

CONTENTS

PREFACE

This monograph contains a selection of papers that were presented at the 38th International Conference *Problems of Building and Estimation of Econometric Models – MACROMODELS 2011*, held jointly with the 16th AMFET Conference on Modelling Economies in Transition in Poznań, Poland between 30 November and 3 December 2011.

The 38th MACROMODELS conference followed a long established tradition of international meetings held initially to discuss the issues of transition from the European perspective, making use of the modelling exercises. Recently, it has become a forum of discussions on more general issues of integration and development in Europe. Also special emphasis was put to the area of financial processes. On the methodological plane the applications of econometric procedures based on non-stationary time series prevailed.

Since their establishment in 1974 the meetings have been organised in different sites of Poland, despite numerous economic and political turbulences. The Conference Proceedings have been published on an annual basis since 1982. The latest editions are available on request.

The meetings offered the floor for discussions about the models' specification and use, and about applied econometric methodology. Initially they were particularly centred on issues in the modelling of the former centrally planned economies and since 1990 on the economies in transition before and after the accession to the European Union.

The specification of equations and their systems, their testing and validations were shown. More recently the issues in the theory of growth and their empirical applications have been presented for discussion. First results based on the DSGE models were announced. In recent years the modelling of financial processes has gained importance.

New estimation methods allowing for changes in economic regimes and structural parameters, the use of extraneous information, as well as the ARCH and GARCH models were presented. The issues were open to a more general discussion about macroeconomic modelling and about the development of econometric methodology; more recently about the use of non-stationary time series analysis, especially vector cointegration, model validation using impulse response functions and the Bayesian methodology.

The participants of these meetings represented in the past mainly the Central and East European and Scandinavian countries. However, there has always been a strong representation of the West European and American scholars, with the Nobel Prize Laureates – Professor Lawrence R. Klein and more recently in Cedzyna (2002) – Professor Robert Engle, the Nobel Prize Laureate in 2003. In Warsaw in 2003 we hosted prominent European scholars as invited speakers, including Professors David Hendry, Stephen G. Hall, Søren Johansen, Katarina Juselius, Helmut Lütkepohl. It should be also stressed that in the last 10 years in MACROMODELS participated also: Anindya Banerjee, M. Hashem Pesaran, Timo Teräsvirta, Peter Winker. MACROMODELS has recently become an important forum for applied econometricians from all over Europe.

The Conference proceedings published in this monograph cover the following topics:

- Macroeconometric Models,
- Financial Econometrics,
- Econometric Methods.

It contains the articles accepted for publication and abstracts of the remaining papers. The English language and aditional verification has been performed by Waldemar Florczak.

We would like to thank the Polish Academy of Sciences (PAN) and the Ministry of Science and Higher Education, the Faculty of Economics and Sociology and the Institute of Econometrics, University of Łódź for their financial support, and our numerous colleagues and the editorial staff for their efforts, which made it possible to publish this monograph.

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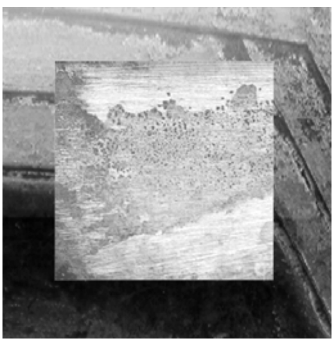
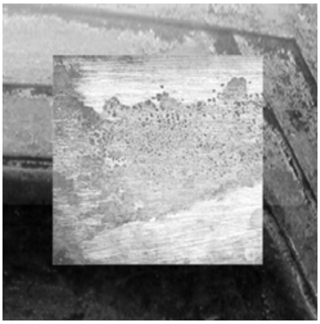
Łódź, November 2012



MACROECONOMETRIC MODELS



MACROMODELS 2011
Łódź 2012, pp. 15–24

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Potential Existence of Devaney, Li-Yorke and Distributional Chaos in two Modifications of Macroeconomic IS-LM Model*

Abstract. In this paper we consider two modifications of the macroeconomic IS-LM model, especially their dynamical behaviour and potential existence of Devaney, Li-Yorke and distributional chaos. First of these models follows the original Keynesian approach of the IS-LM model which is demand-oriented and which considers a supply of money as the exogenous quantity. We add to this model the ideas of Nicholas Kaldor about properties of the investment and saving function. This original model has three deficiencies. These are demand-orientation model, the assumption of constant price level and the exogenous money supply, i.e. the money stock determined only by the Central bank. The modified model eliminates these deficiencies. In this contribution, we describe a behaviour of these models in time, i.e. their stability or unstability. Last but not least, we present how specific types of chaos can exist in these models based on the special type of differential inclusion called the Euler equation branching.

INTRODUCTION AND BASIC NOTATIONS

The first of the considered models is the original Keynesian IS-LM model with general (not linear) investment and saving function and the general function of demand for money. This model accepts the conception of supply of money as exogenous quantity. The model also includes Kaldor's idea of non-linear

* The research was supported, in part, by the Student Grant Competition of Silesian University in Opava, grant no. SGS/19/2010.

investment and saving function with "sigma-shaped" graphs and the Keynesian conception of demand for money.

The second model, called modified IS-LM model, tries to eliminate three main deficiencies of the first model.

In the last section, we research into the stability of these macroeconomic systems and describe how the Euler equation branching (the type of differential inclusion) can originate in these models. Based on this equation, different types of chaos (i.e. Devaney, Li-Yorke and distributional chaos) can exist.

Basic Notation:

t – time,
Y – aggregate income (GDP, GNP),
R – interest rate,
I – investments,
S – savings,
L – demand for money,
M – supply of money.

1. EXTENDED ORIGINAL THE KEYNESIAN IS-LM MODEL

1.1. Assumptions of Extended Original Model

- a two-sector economy,
- $Y \geq 0, R > 0$
- a special conditions of the investment, saving function and of the demand for money function,
- a money supply as an exogenous quantity,
- a demand-oriented model,
- a constant price level (i.e. absence of inflation effect).

1.2. Static and Dynamic Models

We can see the formulation of static and dynamic version of the original Keynesian IS-LM model for example in Gandolfo (1997).

Assumption 1.1.

The original Keynesian static IS-LM model is defined by

$$\begin{aligned} IS &: I(Y,R) = S(Y,R) \\ LM &: L(Y,R) = M_S \end{aligned} \qquad (1)$$

where:

$I(Y, R)$ – is an general investment function,
$S(Y, R)$ – is a general saving function,
$L(Y, R)$ – is a demand for money function,
M_S – is a constant representing supply of money, $M_S > 0$.

Assumption 1.2.

The *original Keynesian dynamic IS-LM model* is defined

$$\begin{aligned} \frac{dY}{dt} &= \dot{Y} = \alpha[I(Y,R) - S(Y,R)] \\ \frac{dR}{dt} &= \dot{R} = \beta[L(Y,R) - M_S] \end{aligned} \tag{2}$$

where $\alpha, \beta > 0$ are parameters of dynamics.

1.3. Properties of this Model's Functions

Assumption 1.3.

The functions I(Y, R), S(Y, R) and L(Y, R) satisfy the economic conditions which are defined in the following way:

$$0 < \frac{\partial I}{\partial Y} < 1, \quad \frac{\partial I}{\partial R} < 0, \quad \frac{\partial^2 I}{\partial R^2} > 0, \tag{3}$$

$$0 < \frac{\partial S}{\partial Y} < 1, \quad \frac{\partial S}{\partial R} > 0, \quad \frac{\partial^2 S}{\partial R^2} < 0, \tag{4}$$

$$\frac{\partial L}{\partial Y} > 0, \quad \frac{\partial^2 L}{\partial Y^2} < 0, \quad \frac{\partial L}{\partial R} < 0, \quad \frac{\partial^2 L}{\partial R^2} > 0. \tag{5}$$

These economic conditions mean that $I(Y)$ (for some fixed R) is an increasing function with relatively small slope and $I(R)$ (for some fixed Y) is a decreasing and convex function.

Then, $S(Y)$ (for some fixed R) is an increasing function with relatively small slope and $S(R)$ (for some fixed Y) is an increasing and concave function.

The last economic condition relates to the function of demand for money. The function of demand for money is the sum of the transactional and the speculative demand for money. The function of the transactional demand for money depends only on aggregate income, let denote $L_1(Y)$, and the function of the speculative demand for money depends only on interest rate, let denote

$L_2(R)$. $L_1(Y)$ is an increasing and concave function and $L_2(R)$ a decreasing and convex function. This corresponds to the economic properties of investment, savings or money demand functions according to the Keynesian theory.

The following conditions, i.e. Kaldor's conditions and the sufficient conditions of existence of at least one intersection point of the IS curve and of the LM curve, were formulated in Baráková (2004).

Assumption 1.4.

So-called "Kaldor's conditions" are

$$S_Y > I_Y \quad \text{for} \quad Y \in (-\infty, M)$$

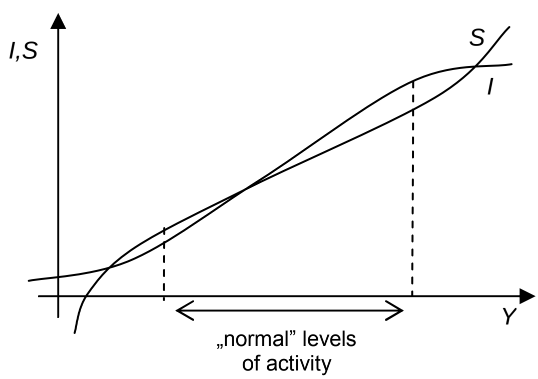
$$S_Y < I_Y \quad \text{for} \quad Y \in (M, N)$$

$$S_Y > I_Y \quad \text{for} \quad Y \in (N, \infty) \qquad (6)$$

where points $M < N$ are given by equation $I_Y(Y,R) = S_Y(Y,R)$ for some fixed R.

Graphs of the functions I(Y) and S(Y) should have so-called "sigma-like" shape according to Kaldor. For so-called "normal" levels of activity the investment and savings functions are getting near the linear function.

Figure 1. The graphs of $I(Y)$ and $S(Y)$



Assumption 1.5.

The *sufficient conditions of existence of at least one intersection point* of the IS curve and of the LM curve are

- for some fixed $Y \in (-\infty, \infty)$

  $\lim_{R\to\infty}[I(Y,R) - S(Y,R)] = -\infty$

$\lim_{R\to-\infty}[I(Y,R)-S(Y,R)]=\infty$ (7)

- for some fixed $R\in(-\infty,\infty)$

$\lim_{Y\to\infty} L(Y,R)=\infty$

$\lim_{R\to-\infty} L(Y,R)=-\infty$ (8)

2. MODIFIED IS-LM MODEL

In this section, there is a definition of a new modified IS-LM model that is free from the three main deficiencies of the original IS-LM model being supposition of constant price level, strictly exogenous money supply and strictly demand-orientation. This section also contains the assumptions of the modified model.

2.1. Elimination of the Three Deficiencies of the Original Model

2.1.1. Elimination of Constant Price Level

We add to our model a floating price level, i.e. inflation effect. We need to distinguish three type of interest rate:

R – long-term real interest rate,
i_S – short-term nominal interest rate,
i_L – long-term nominal interest rate.

There is the long-term real interest rate on goods market and the short-term nominal interest rate on money market (or financial assets market). It holds a few well-known relations:

$i_L=i_S+MP,$

$R=i_L-\pi^e,$

$i_S=R-MP+\pi^e,$ (9)

where
MP – maturity premium,
π^e – inflation rate.

2.1.2 Elimination of Strictly Exogenous Money Supply

We consider that the money supply is not strictly exogenous quantity (i.e. a money stock determined by central bank), but the supply of money is endogenous quantity (i.e. money is generated by credit creation) with some certain money determination by central bank.

Assumption 2.1.
We define the function of money supply by the following formula:

$$M = M(Y, i_S) + CB \tag{10}$$

where
$M(Y, i_S)$ – is a general function of money supply depending on Y and i_S,
CB – is a constant representing interventions of the central bank, $CB > 0$.

Assumption 2.2.
The economic properties of money supply function are:

$$0 < \frac{\partial M}{\partial Y} < \frac{\partial L}{\partial Y}$$

$$\frac{\partial M}{\partial i_S} > 0, \ \frac{\partial^2 M}{\partial {i_S}^2} < 0. \tag{11}$$

The first formula means that the relation between supply of money and aggregate income is positive and that the rate of increase of money supply depending on aggregate income is smaller than the rate of increase of money demand depending on aggregate income because the banks are more cautious than other agents. And the second formula means that the relation between supply of money and interest rate is positive and that the rate of increase of money supply depending on interest rate is decreasing.

2.1.3 Elimination of Strictly Demand-Oriented Model

Demand-oriented model means that supply is fully adapted to demand. Vice-versa supply-oriented model means that demand is fully adapted to supply. In fact the real situation is the conjunction of demand-oriented model and supply-oriented model.

We can model this situation by using the so-called Euler equation branching. This is the type of differential inclusion. More precisely

Assumption 2.3.
The Euler equation branching is

$$\dot{x} \in \{f(x), g(x)\}, \tag{12}$$

where f, g: $X \rightarrow R^n$ are continuous, $X \in R^n$ is open and it holds $f(x) \neq g(x)$ for each $x \in X$.

The first "branch" (i.e. $f(x)$) represents the demand-oriented model and the second "branch" (i.e. $g(x)$) represents the supply-oriented model. Then we can search for solutions of this model represented by these two branches (i.e. solutions of this type of differential inclusion) and investigate its stability. The solutions may involve “switching” between the branches *f* and *g*.

2.2 Assumptions of the Modified Model

- two-sector economy,
- $Y \geq 0,\ R \in R^1$,
- a special conditions of the investment, savings function and of the function of demand for money,
- a money supply as a conjunction of endogenous and exogenous quantity,
- a conjunction of demand-oriented and supply-oriented model,
- a variable price level (i.e. included inflation effect).

2.3. Formulation of the Modified Model

Assumption 2.4.

The demand-oriented modified static IS-LM model is defined by

$$IS : I(Y,R) = S(Y,R)$$

$$IM : L(Y, R - MP + \pi^e) = M(Y, R - MP + \pi^e) + CB \quad (13)$$

Assumption 2.5.

The demand-oriented modified dynamic IS-LM model is defined by

$$\frac{dY}{dt} = \alpha[I(Y,R) - S(Y,R)]$$

$$\frac{d(R - MP + \pi^e)}{dt} = \beta[L(Y, R - MP + \pi^e) - M(Y, R - MP + \pi^e) - CB] \quad (14)$$

The functions $I(Y, R)$, $S(Y, R)$, $L(Y, i_s)$, $M(Y, i_S)$ (where $i_S = R - MP + \pi^e$) have the same properties, which are described above.

Remark 2.1.

The *sufficient condition of existence of at least one intersection point* of the IS curve and of the LM curve holds in the modified form:

for some fixed $Y \in (-\infty, \infty)$

$$\lim_{R\to\infty}[I(Y,R) - S(Y,R)] = -\infty$$

$$\lim_{R\to-\infty}[I(Y,R) - S(Y,R)] = \infty \quad (15)$$

for some fixed $R \in (-\infty, \infty)$

$$\lim_{R\to\infty}[L(Y,R-MP+\pi^e)-M(Y,R-MP+\pi^e)-CB]=\infty$$
$$\lim_{R\to-\infty}[L(Y,R-MP+\pi^e)-M(Y,R-MP+\pi^e)-CB]=-\infty \qquad (16)$$

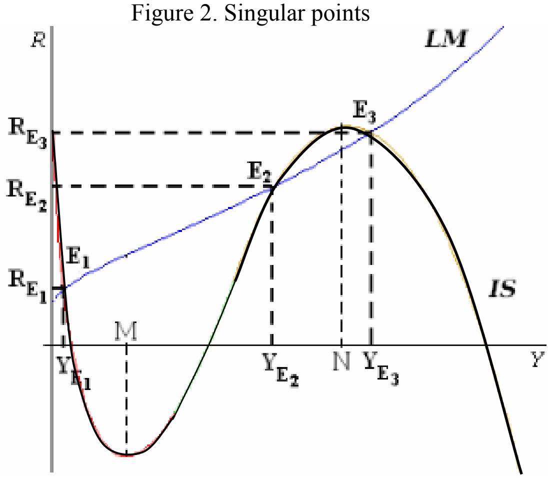
Now, we need *supply-oriented modified static IS-LM model and supply-oriented modified dynamic IS-LM model*. These two models must be created. The IS equation will be different (maybe the name IS will not be apposite). The demand side will represent only the level of aggregate income (in demand oriented model it is supply side). The supply side will perhaps be based on some modification of a production function. The LM equation will be very similar to the original LM equation.

3. DYNAMICAL BEHAVIOUR AND EXSTENCE OF CHAOS

This section contains description of dynamic behaviour of our models and explains how Devaney, Li-Yorke or distributional chaos can originate in these models.

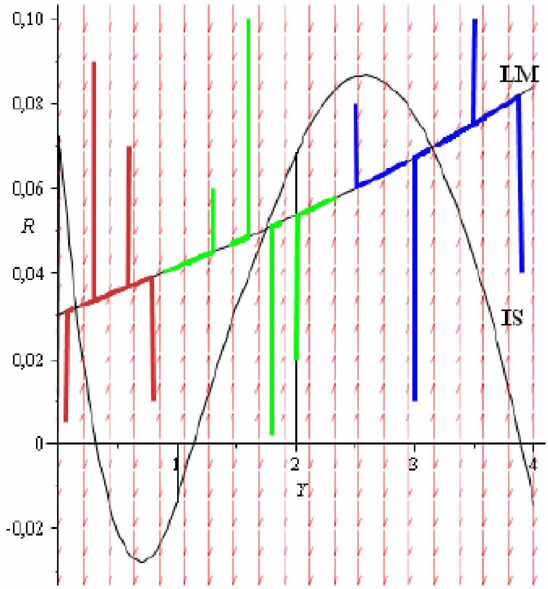
Dynamic behaviour can be described using the singular points. The curves IS or LM (also called isoclines) represent the stability on the goods market, or on the money market (or financial assets market).

Figure 2. Singular points



In the both models there can exist at most 3 singular points. The both points on the edge denoted E_1 and E_3 are stable node and the middle singular point denoted E_2 is an unstable saddle point. We can see these types of singular points in the following figure.

Figure 3. Phase portrait



This phase portrait displays characterisation of the behaviour of the economy.

I would like to note some special behaviour of this example. First, the economy converges to the stability on the money market (so to the LM curve). Then the economy converges to the aggregate macroeconomic equilibrium point along the LM curve.

The three types of chaos, i.e. Devaney, Li-Yorke and distributional chaos, can originate if there exist Euler equation branching.

The first "branch" (i.e. $f(x)$) is the extended original Keynesian dynamic IS-LM model or demand-oriented modified dynamic IS-LM model and the second "branch" (i.e. $g(x)$) is some differential equation represented by the supply-oriented model.

According to Stockman et alia (2010), the following theorem holds:

Theorem 3.1.

Let $x^* \in X$, $f(x^*) = 0$ and $g(x^*) \neq 0$, $A = Df(x^*)$ with eigenvalues λ_1, λ_2 and correspondent eigenvectors e_1, e_2. We choose $\delta > 0$ such that $g(x) \neq 0$ for every $x \in \overline{B}_\delta(x^*)$. Let the solution of $\dot{x} = g(x)$ be no bounded in some subset of R^2.

1) We assume that there exist $\varepsilon > 0$ such that x^* is source (i.e. unstable focus) or sink (i.e. stable focus) for $x \in B_\varepsilon(x^*)$. Then there exists Devaney, Li-Yorke and distributional chaos.

2) We assume that $\lambda_1 < 0$, $\lambda_2 > 0$ (i.e. x^* is saddle point) and $g(x^*) \neq \alpha e_1$, $g(x^*) \neq \beta e_2$, where $\alpha, \beta \in R$. Then there exist Devaney, Li-Yorke and distributional chaos.

Our considered models have a singular point in the middle which fulfils the previous theorem, meaning it is the saddle point. If there is no-bounded solution in some subset of R^2 in our supply-oriented "branch", there will originate Devaney, Li-Yorke and distributional chaos.

In the end, we can speculate about different view on this problem. We can formulate a hypothesis that if both of our branches, i.e. $\dot{x} = f(x)$ and $\dot{x} = g(x)$, have some periodic solution (i.e. bounded solution), then there will exist Li-Yorke chaos but three will not exist distributional chaos.

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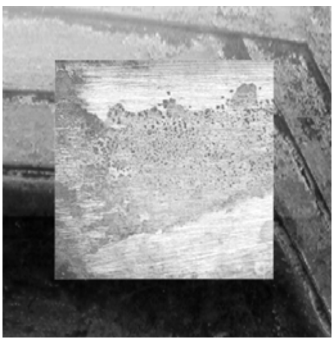
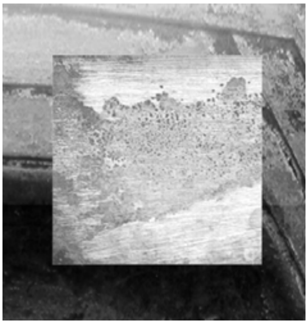


ABSTRACTS

MACROMODELS 2011
Łódź 2012, p. 27



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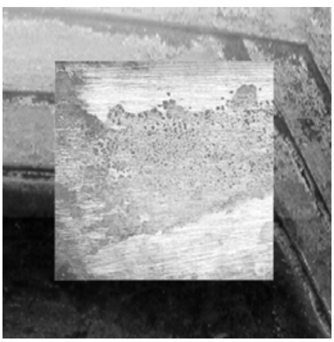
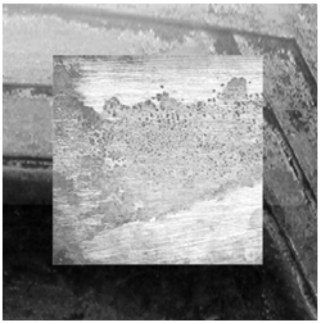
The Role of Endogenous Government Spending in the Hicksian Model with Investment Floor and Income Ceiling

Abstract. We investigate the dynamics of the nonlinear Hicks-type model with investment floor, income ceiling and endogenous government spending. We analyse the impact of government spending on the dynamics of the domestic product. Particular attention is placed on the impact of induced government spending on the dynamics of the model. We determine the equilibria and investigate their local asymptotic stability and investigate how the dynamics of the model depends on the parameters. We present analytical results whenever it is possible and numerical simulations of the more interesting occurrences.



FINANCIAL ECONOMETRICS



MACROMODELS 2011
Łódź 2011, pp. 31–42

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Linear Combinations of Volatility Forecasts for the WIG20 and Polish Exchange Rates

Abstract. As is known forecast combinations may be better forecasts then forecasts obtained with single models. The purpose of the research is to check if linear combination of forecasts from models for of the WIG20 Index and different currency exchange rates is a good solution when searching for the best forecasts. We check if the forecasting models are highly correlated with response variable and poorly correlated with each other so if they fulfill the Hellwig assumptions.

Key words: volatility, forecasts, linear regression, MCS
JEL classification: C52, C53

1. INTRODUCTION

According to Stock and Watson (2004) the combination of the models generates better forecast then the single model. A combination of forecasts is a good choice when it is not possible to distinguish one dominant model (Timmermann 2006). Another argument for a combination is that the combinations of forecasts are more stable then individual forecasts (Stock, Watson 2004).

The aim of the paper is to verify if linear combination of forecasts of volatility for WIG20 and different exchange rates are a good solution when searching for best volatility forecasting models. We check if the forecasts are highly correlated with response variable and poorly correlated with each other so if they fulfill the “Hellwig assumptions”. We compare the volatility of forecasts with daily realized volatility. We investigate the results for different measures of realized volatility and different best forecasting models for different functions of error.

2. FORECASTS COMBINATIONS

The simplest combination is a linear one with the identical coefficients and the sum of the weights equal one.

$$g\left(\hat{y}_{t+h};\omega_{t+h,t}\right)=\frac{1}{N}\sum_{j=1}^{N}\hat{y}_{t+h,t,j} \tag{1}$$

where $\hat{y}_{t+h,t}$ is the forecast, and $\omega_{t+h,t}$ is the weight.

The forecast error is defined by:

$$e^{c}_{t+h,t}=y_{t+h}-g\left(\hat{y}_{t+h,t};\omega_{t+h,t}\right) \tag{2}$$

The parameters of the optimal combination of the forecasts are in this case the solution of the following problem

$$\omega^{*}=\underset{\omega\ \in W_t}{\arg\min}\ E\left[L\left(e^{c}(\omega)\right)\right] \tag{3}$$

where L denotes mean squared error (MSE) loss.

Under MSE the combination weights depend only on the first two moments of the joint distribution of y_{t+h} and $\hat{y}_{t+h,t}$

$$\begin{pmatrix} y_{t+h} \\ \hat{y}_{t+h} \end{pmatrix} \sim \begin{pmatrix} \mu_{y_{t+h,t}} \\ \mu_{\hat{y}_{t+h,t}} \end{pmatrix} \begin{pmatrix} \sigma^{2}_{y_{t+h,t}} & \sigma'_{y\hat{y}_{t+h,t}} \\ \sigma_{y\hat{y}_{t+h,t}} & \Sigma_{\hat{y}\hat{y}_{t+h,t}} \end{pmatrix} \tag{4}$$

For MSE Timmermann (2006) obtained the following optimal weights:

$$\omega_0^{*}=\mu_{y_{t+h,t}}-\omega^{*}\mu_{\hat{y}_{t+h,t}},\ \omega^{*}=\Sigma^{-1}_{\hat{y}\hat{y}_{t+h}}\sigma_{y\hat{y}_{t+h,t}} \tag{5}$$

Consider the combination of two forecasts $\hat{y}_1, \hat{y}_2$. Let e_1 i e_2 denote the forecast errors. Assume $e_1 \sim \left(0,\sigma_1^2\right)$, $e_2 \sim \left(0,\sigma_2^2\right)$, where $\sigma_1^2=Var\left(e_1\right)$, $\sigma_2^2=Var\left(e_2\right)$ and $\sigma_{12}=\rho_{12}\sigma_1\sigma_2$ is the covariance between e_1 and e_2, whereas ρ_{12} is their correlation.

The optimal weights for this combination by Timmermann (2005) have the form

$$\omega^* = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}},\ 1-\omega^* = \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}^2}\ . \tag{6}$$

The identical weights are optimal if the forecast variances are the same and independent of the correlation between forecasts providing the forecasts are unbiased (Timmermann 2006). The natural example is the following scheme of two forecasts:

$$(1/2)\cdot(\hat{y}_1 + \hat{y}_2). \tag{7}$$

When the forecast are unbiased Timmermann (2006) proposes the combined that gives the inverse weights to the forecasts with the assumption that the correlation is zero:

$$\omega_{inv} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2},\ 1-\omega_{\text{inv}} = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}. \tag{8}$$

For N forecasts one can assume $0 \le \omega_{ni} \le 1,\ i = 1,\dots,N$ to make the values of the combination forecasts be in the interval of values of the individual forecasts. Let $\hat{y}_c = \omega\hat{y}_1 + (1-\omega)\hat{y}_2$, $y - \hat{y}_1 = e_1 \sim (0, \sigma^2)$, $y - \hat{y}_2 = e_2 \sim (\mu_2, \sigma^2)$, so $\hat{y}_2$ is the biased forecast and assume $\text{cov}(e_1, e_2) = \sigma_{12} = \rho_{12}\sigma^2$.

Using the formulas

$$e_c = \omega e_1 + (1-\omega)e_2$$

$$\sigma_c^2(\omega) = \omega^2\sigma^2 + (1-\omega)^2(\sigma^2 + \mu^2) + 2\omega(1-\omega)\sigma_{12}\,.$$

Timmermann obtained

$$MSE(\hat{y}_c) - MSE(\hat{y}_1) = (1-\omega)\sigma^2\left((1-\omega)\left(\frac{\mu_2}{\sigma}\right)^2 - 2\omega(1-\rho_{12})\right) \tag{9}$$

So if $\left(\frac{\mu_2}{\sigma}\right)^2 > \frac{2\omega(1-\rho_{12})}{1-\omega^2}$, then $MSE(\hat{y}_c) > MSE(\hat{y}_1)$.

The condition always holds for $\rho_{12} = 1$. In this case the forecast of the combination of models doesn't outperform the unbiased forecast of the simple model. What is more, the bigger the bias of the forecast is, the smaller the advantage of the combinations. If the forecasts are biased then identical weights are optimal when the forecast errors have the same variance and identical correlation between forecasts (Timmermann 2006) .

The optimal weights problem may be formulated as the optimalization task of minimalization of the expected forecast error variance $\Sigma_e = E[ee']$, where $e = y - l\hat{y}$ providing that the sum of weights is one and the individual forecasts are unbiased:

$$\min \omega' \Sigma_e \omega. \tag{10}$$

$$\omega' l = 1. \tag{11}$$

where l is the vector of ones.

For the invertible covariance matrix Σ_e Timmerman (2006) obtains the following optimal weights:

$$\omega^* = \left(l' \Sigma_e^{-1} l\right)^{-1} \Sigma_e^{-1} l. \tag{12}$$

The problem of the optimal combination can be solved as the following test

$$\begin{aligned} H_0 &: E\left[L\left(\hat{\sigma}_t^2, h_t^A\right)\right] = E\left[L\left(\hat{\sigma}_t^2, f\left(h_t^A, h_t^B, \theta\right)\right)\right] \\ H^A &: E\left[L\left(\hat{\sigma}_t^2, h_t^A\right)\right] > E\left[L\left(\hat{\sigma}_t^2, f\left(h_t^A, h_t^B, \theta\right)\right)\right] \end{aligned} \tag{13}$$

The test statistic of Diebold-Mariano and West (DMW) can be used in the test. Let define the difference

$$d_t = L\left(\hat{\sigma}_t^2, h_t^A\right) - L\left(\hat{\sigma}_t^2, f\left(h_t^A, h_t^B, \theta\right)\right) \tag{14}$$

Then the DMW test statistic is the following:

$$DMW_T = \frac{\sqrt{T} d_T}{\sqrt{a\hat{V}ar[Td_t]}}, \tag{15}$$

where

$$\bar{d}_T \equiv \frac{1}{T}\sum_{t=1}^{T} d_t. \qquad (16)$$

Under the null hypothesis the test statistic has normal distribution.
If

$$\sigma(y-\hat{y}_1) > \sigma(y-\hat{y}_2) \qquad (17)$$

$$\operatorname{cov}(y-\hat{y}_1, y-\hat{y}_2) \neq \sigma(y-\hat{y}_2)\sigma(y-\hat{y}_1), \qquad (18)$$

the optimal model is the combination of forecasts, Timmermann (2006).

Another scheme can be created on the basis of the ranking of models by Aiolfi and Timmermann (2006). Let R_i be the position of the *i*-model in the ranking. The weights of the combination are the following:

$$\hat{\omega} = R_i^{-1} / \left(\sum_{i=1}^{N} R_i^{-1}\right). \qquad (19)$$

3. HELLWIG'S IDEA

In good linear regression model:

1) explanatory variables are highly correlated with response variable,
2) explanatory variables are poorly correlated with each other.

What is more, big correlations between explanatory variables cause big average errorsin parameters.

4. DATA

In the empirical investigation we used daily observations of the WIG20 Index, from May 8, 2001 till May 8, 2009 for the model estimation. On the next 256 data from 29 April 2008 till 8 May, 2009 we calculated 1 day volatility forecasts. To evaluate the quality of our forecasts we compared them with daily realized volatility calculated for 5, 10 and 30 minute intraday returns.

We considered the following types of GARCH (1, 1) with different distributions of error: RiskMetrics, GARCH, EGARCH, GJR, APARCH, IGARCH, FIGARCH-BBM, FIGARCH-CHUNG, FIEGARCH, FIAPARCH-

BBM, FIAPARCH-CHUNG, HYGARCH. The models were estimated with different distributions of error: GAUSSIAN, STUDENT-t, and GED, SKEWED – STUDENT

5. THE REALIZED VOLATILITY

The realized volatility can be calculated by summing the squares of intraday returns. With the use of an equation which allows for the night return it is defined as follow:

$$\sigma^2_{2,t} = \sum_{i=0}^{N} r^2_{t,i}, \tag{20}$$

where the intraday return on day n and in the moment d is :

$$r_{n,d} = 100(\ln P_{n,d} - \ln P_{n,d-1}),\ r_{n,0} = 100(\ln P_{n,1} - \ln P_{n-1,N}), \tag{21}$$

N is the number of periods in a day.

The alternative approach was proposed by Andersen and Bollerslev (1997). They suggested representing the daily volatility as the sum of intraday returns

$$\sigma^2_{1,t} = \sum_{i=1}^{N} r^2_{t,i}. \tag{22}$$

They suggest multiplying $\sigma^2_{1,t}$ by $(1+c)$, where c is the positive constant (Martens 2002). They choose $\left(\sigma^2_{\mathrm{co}} + \sigma^2_{\mathrm{oc}}\right)/\sigma^2_{\mathrm{oc}}$ as the constant c, where $\sigma^2_{\mathrm{co}} = Var(r_{t,0})$ and $\sigma^2_{\mathrm{oc}} = Var(\sum_{t=1}^{N} r_{t,n})$, Koopman et al. (2005). Then the realized volatility can be expressedes:

$$\sigma^2_{3,t} = \frac{\sigma^2_{\mathrm{oc}} + \sigma^2_{\mathrm{co}}}{\sigma^2_{\mathrm{oc}}} \sum_{i=1}^{N} r^2_{t,i} \tag{23}$$

In the article MSE means the mean squared error and MAD means mean absolute deviation, where N is the number of forecasts.

$$\text{MSE} = N^{-1}\sum_{t=1}^{N}\left(\sigma_{l,t}^2 - \hat{\sigma}_{k,t}^2\right)^2, \tag{24}$$

$$\text{MAD}= N^{-1}\sum_{t=1}^{N}\left|\sigma_{l,t}^2 - \hat{\sigma}_{k,t}^2\right|, \tag{25}$$

where $l \in \{1, 2, 3\}$, $k \in \{1, \ldots, m\}$ is the number of models from the considered set. In the following formula $\hat{\sigma}_{k,t}^2$ is the forecast of volatility from the model k on the moment *t*, $\sigma_{l,t}^2$ is the value of the realized volatility of the type *l* in the moment *t.*

6. EMPIRICAL RESULTS

The best models obtained with Model Confidence Set method (MCS) for MAD loss function, realized volatility $\sigma_{1,t}^2$, $\sigma_{2,t}^2$, $\sigma_{3,t}^2$ and 5 minute frequency of returns are:

1. GARCH (1,1) with Gaussian distribution of error
2. AR(1)-GARCH with Gaussian distribution of error
3. MA(1)-GARCH with Gaussian distribution of error
4. HYGARCH with Gaussian distribution of error
5. AR(1)-HYGARCH with Gaussian distribution of error
6. MA(1)-HYGARCH with Gaussian distribution of error

The matrix of correlations:

Table 1. The values of correlations between forecasts

	1	2	3	4	5	6
1	1	0.999678	0.999691	0.985923	0.98323	0.983293
2		1	0.999999	0.984996	0.982814	0.982866
3			1	0.984982	0.982782	0.982836
4				1	0.999253	0.999267
5					1	0.999999
6						1

The bests model obtained with MCS method for MSE loss function, realized volatility $\sigma_{1,t}^2$, $\sigma_{2,t}^2$, $\sigma_{3,t}^2$ and 5 minute frequency of returns is:

RiskMetrics with skewed Student t distribution of error.

The MCS for MAD, realized volatility $\sigma_{1,t}^2$, $\sigma_{3,t}^2$ and 10 minute frequency of returns is:

1. GARCH (1,1) with Gaussian distribution of error
2. AR(1)-GARCH with Gaussian distribution of error
3. MA(1)-GARCH with Gaussian distribution of error
4. HYGARCH with Gaussian distribution of error

The matrix of correlations is as follows:

Table 2. The values of correlations between forecasts

	1	2	3	4
1	1	0.999678	0.999691	0.985923
2		1	0.999999	0.984996
3			1	0.984982
4				1

The MCS for MSE, realized volatility $\sigma_{1,t}^2$, $\sigma_{2,t}^2$, $\sigma_{3,t}^2$ and 10 minute frequency of returns is:

1. FIGARCH with GED (Gaussian distribution of error)
2. AR(1)-RiskMetrics with Gaussian distribution of error
3. RiskMetrics with skewed Student distribution of error
4. GARCH with skewed Student – t distribution of error

The matrics of correlations is as follows:

Table 3. The values of correlations between forecasts

	1	2	3	4
1	1	0.985936	0.987563	0.987633
2		1	0.99972	0.995763
3			1	0.996994
4				1

The best models obtained with MCS method for MAD loss function, realized volatility $\sigma_{1,t}^2$, $\sigma_{3,t}^2$ and 30 minute frequency of returns are:

1. GARCH (1,1) with Gaussian distribution of error
2. AR(1)-GARCH with Gaussian distribution of error
3. MA(1)-GARCH with Gaussian distribution of error
4. HYGARCH with Gaussian distribution of error
5. AR(1)-HYGARCH with Gaussian distribution of error

6. MA(1)-HYGARCH with Gaussian distribution of error

The matrix of correlations is as follows:

Table 4. The values of correlations between forecasts

	1	2	3	4	5	6
1	1	0.999678	0.999691	0.985923	0.98323	0.983293
2		1	0.999999	0.984996	0.982814	0.982866
3			1	0.984982	0.982782	0.982836
4				1	0.999253	0.999267
5					1	0.999999
6						1

The best models obtained with MCS method for MAD loss function, realized volatility $\sigma^2_{2,t}$ and 30 minute frequency of returns are:

1. GARCH with GED

2. FIGARCH with GED

3. ARMA(1,1) – GARCH with GED

4. GARCH with skewed Student t

The matrics of correlations:

Table 5. The values of correlations between forecasts

	1	2	3	4
1	1	0.98712	0.999998	0.999795
2		1	0.987216	0.987633
3			1	0.999815
4				1

The MCS for MSE, realized volatility $\sigma^2_{1,t}$, $\sigma^2_{2,t}$, $\sigma^2_{3,t}$ and 30 minute frequency of returns is:

1. AR(1) – RiskMetrics with Gaussian distribution of error

2. RiskMetrics with skewed Student t distribution of error

The matrics of correlations are:

Table 6. The values of correlations between forecasts

	1	2
1	1	0.99972
2		1

7. THE ESTIMATES OF THE PARAMETERS OF THE BEST MODELS

Table 7. The estimates of the parameters of the best models

Model	GARCH	AR(1)-GARCH	MA(1)-GARCH	HYGARCH	AR(1)-HYGARCH	MA(1)-HYGARCH
Distribution / Parameters	Gauss	Gauss	Gauss	Gauss	Gauss	Gauss
μ				0.07171 (0.03309)	0.07254 (0.03499)	0.07262 (0.03486)
a_1		0.05284 (0.02422)			0.05588 (0.02349)	
b_1			–0.05239 (0.02398)			–0.05554 (0.02325)
ω	0.07077 (0.0562)	0.06925 (0.0542)	0.06933 (0.05242)	0.2277 (0.1327)	0.23065 (0.1292)	0.23096 (0.1294)
α_1	0.06009 (0.0132)	0.06007 (0.01281)	0.06006 (0.01281)	–0.050178 (0.09521)	–0.50707 (0.08936)	–0.50696 (0.08956)
β_1	0.90831 (0.03599)	0.90895 (0.03475)	0.90892 (0.03477)	0.68262 (0.10113)	0.6759 (0.09717)	0.67568 (0.09744
k				0.86847 (0.0712)	0.87852 (0.0741)	0.87831 (0.0713)
d				0.59709 (0.0658)	0.5981 (0.0697)	0.59698 (0.066)

Table 8. The estimates of the parameters of the best models

Model	GARCH(1,1)	FIGARCH(1,d,1)	ARMA(1,1) - GARCH(1,1)	GARCH(1,1)
1	2	3	4	5
Distribution / Parameters	GED	GED	GED	skewed – Student t
μ				0.06983 (0.03343)
a_1			0.78602 (0.12383)	
b_1			0.78602 (0.13456)	
ω	0.01502 (0.0048)	0.08545 (0.095)	0.04616 (0.0259)	0.03765 (0.019)
α_1		–0.47563 (0.23607)	0.05527 (0.00976)	0.05744 (0.00992)

Table 8 (cont.)

1	2	3	4	5
β_1		0.72345 (0.17126)	0.9252 (0.01786)	0.92862 (0.01394)
d		0.50567 (0.2454)		
ν	1.35756 (0.0838)	1.38342 (0.0721)	1.40532 (0.0783)	7.51165 (1.402)
ξ				1.04176 (0.0305)

Table 9. The estimates of the parameters of the best models

Model	RiskMetrics	AR(1)- RiskMetrics	GARCH(1,1)
Distribution / Parameters	skewed - Student *t*	Gauss	skewed – Student *t*
μ	0.06906 (0.03287)		0.06983 (0.003343)
a_1		0.05468 (0.02302)	
ω	0.016 (0.005)		0.03765 (0.019)
α_1			0.05744 (0.00992)
β_1 or λ		0.94	0.92862 (0.01394)
ν	6.7712 (1.2878)		7.51165 (1.4009)
ξ	1.04346 (0.0305)		1.04176 (0.0305)

8. CONCLUSION

W conclude that linear combination of volatility forecasts doesn't outperform the forecast from the single model, because of the big correlations between forecasts for the WIG20 Index. The conclusion the same for main Polish exchange rates volatility forecasts, not presented in the article.

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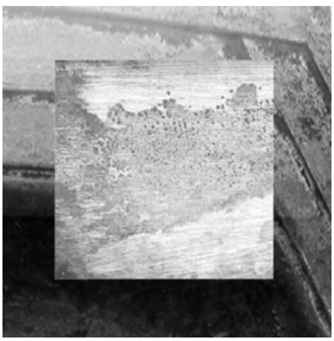
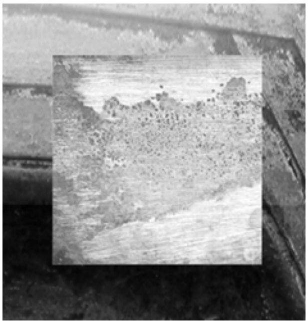


ABSTRACTS



MACROMODELS 2011
Łódź 2012, p. 45

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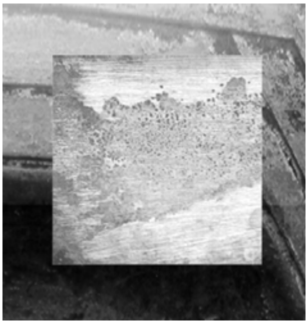
American Versus German Macro Announcements: The Comparison of the Intraday Effects on the German and the French Stock Markets

Abstract. A growing literature has documented the significance of macroeconomic news announcements in price formation process. Routinely the announcements considered in the literature are from the US. This paper is aimed to compare which announcements, from the US or Germany, have a stronger influence on intraday returns and volatility of the French and the German stock markets. We use flexible Fourier form framework to model intraday series and consider standardized surprises for announcements as well as dummy variables standing for the announcements. The volatility response is examined at short response horizon of 1 hour.



MACROMODELS 2011
Łódź 2012, p. 46–47

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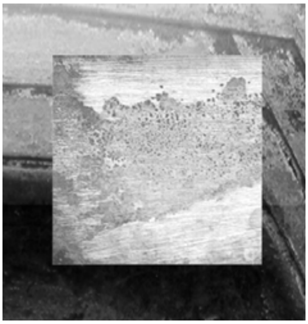
Bayesian Analysis of the ACD Models for Financial UHF Data: Some Specifications and Empirical Results

Abstract: The financial ultra-high frequency data (UHF data) called transaction data or tick-by-tick data is defined to be a full record of transactions and their associated characteristics. The ultra-high frequency transaction data contain two types of observations. One is the time of transaction. The other is a vector of the quantities called the marks, observed at the time of transaction. The marks include price, volume and other characteristics. The most important quality of UHF financial data is the nonsynchronous i.e. erratic distribution of observations over time units. The time intervals between subsequent transactions called trade durations can bring important content about the intensity of the information flow to the market. Duration analyses may furnish information on the microstructure of the financial market, affording a more accurate insight into various market interdependencies. In recent years, much popularing in modeling the durations between the selected events of the transaction process received autoregressive conditional duration models (the ACD models).

The aim of the paper is to present some specifications of the ACD models. Different specifications of the ACD models will be considered and compared with particular emphasis on asymmetric and logarithmic ACD models. In addition, various distributions of innovation term (the Burr distribution and the generalized Gamma distribution) will be considered in the analysis. Bayesian inference will be presented and

practically used to estimation and prediction of the ACD models describing financial ultra-high frequency time series. The empirical part of work will include modeling of trading activity of selected equities from the Warsaw Stock Exchange.



MACROMODELS 2011
Łódź 2012, p. 48–49

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Bayesian Regime Switching SV Models in Market Risk Evaluation

Abstract: The research aims at providing some insight into practical utility of Markov switching stochastic volatility models (or MSSV, in short). Specifically, their forecast abilities in terms of Value at Risk (VaR) and Expected Shortfall (ES) prediction are investigated.

Two classes of the MSSV models are considered. The first one is comprised of a basic stochastic volatility structure with a switching process introduced into parameters of the log-volatility equation. Going beyond the current state of the literature, usually allowing for discrete changes of the volatility intercept solely, the study also features models with shifts in the autoregression coefficient as well as the variance of the volatility error term. As implied by the conditional (upon the current state of the system) moments structure such specifications enable the model to differentiate the states of the market not only in terms of the mean volatility level, but also the variability of the volatility process.

Second class of the MSSV models to be investigated in the paper consists of the SV-in-Mean (SV-M) processes featuring breaks in the risk premium parameter. The SV processes with Markov switching in-Mean effect (or SV-MS-M, in short) has been introduced in our previous work, motivated by somewhat baffling empirical results presented in a large body of literature on the risk-return relationship (see Kwiatkowski 2010). Although far from resolving the ongoing dispute on the very existence of a risk premium in financial markets, our further, yet unpublished research indicated that the SV-MS-M specifications may provide a valid tool of capturing the outliers, which justifies taking

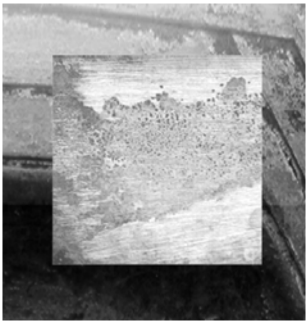
an interest in the SV-MS-M models also in the present context of financial risk analysis.

Much in the vein of a prevalent approach in the literature, switches of the parameters in both of the aforementioned model classes are governed by a homogenous and ergodic Markov chain. The analysis is constrained to two- and three-state specifications only. For a natural purpose of alternative models comparison, apart of regime changing SV structures, simpler, non-switching models (including basic SV and SV-M models) are also of interest.

As regards the estimation and prediction methodology we resort to Bayesian framework, equipped with Markov Chain Monte Carlo simulation tools. The Bayesian approach provides a natural way of tackling uncertainty inherent in model parameters and latent variables (conditional variances and Markov chain states), and allows a formal model comparison with respect to the data fit.

The latter aspect is then extended to the out-of-sample model performance. Particularly, we address the issue of whether the in-sample superiority of the regime switching SV structures (indicated by the marginal data density) translates into superior market risk assessments. To this end two most popular risk measures, Value at Risk and Expected Shortfall, are calculated and compared across different model specifications. Particularly, a wide range of criteria (mainly loss functions) are employed to backtest the series of one- to ten-day ahead VaR and ES forecasts, generated at several tolerance levels for both the long and short positions. In our setting Value at Risk and Expected Shortfall are defined as a quantile and tail conditional mean, respectively, in the marginal predictive distribution of a future simple rate of return. Therefore predictive risk measures, contrary to their non-Bayesian counterparts, take into account the uncertainty featured by all the unknown quantities of the model.

MACROMODELS 2011
Łódź 2012, p. 50



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Detecting Risk Transfer at Financial Markets Using Different Risk Measures*

Abstract. The purpose of the paper is to analyze the process of transferring risk across different financial markets in the global world. It is particularly important in the framework of the current economic crisis. The applied methodology concerns so called Granger-causality in risk defined by Hong in 2001. The statistical procedure was applied over different risk measures, such as value at risk, expected shortfall as well as spectral risk measure. Particular stress was put onto the role of Chinese financial processes in the transferring of risk. It is also shown that the presented procedure indirectly allows evaluating robustness of the risk measures in empirical applications.

Key words: Granger-causality in risk, Chinese financial market, risk transfer, risk measures.

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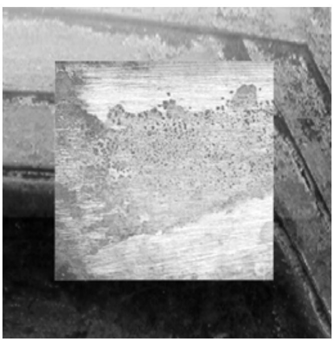
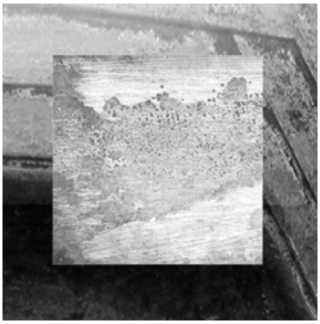


ECONOMETRIC METHODS



MACROMODELS 2011
Łódź 2012, pp. 53–72

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Time-Varying Distributed Lag Models in the Flow Systems

Abstract. The paper analyzes systems composed of flows between certain stocks, where the basic element comprises a stock and flows respectively feeding (inflow) and exhausting that stock (outflow), the latter being described using the time-varying distributed lag model with the inflow rate being the independent variable. Such elements are common in models used to describe consumer demand, levels of gross or net fixed assets, bank loans outstanding, as well as in demography, etc. The concept of the time-varying distributed lag models in this paper is associated with changes of the lag distribution which can be caused by seasonality, or evolving behavior/preferences of economic agents. The paper presents also the properties of complex structures which are the result of summating and/or superposition of the distribution lag models within the flow systems. In particular, parameters of lag distribution, such as mean lag and variance of such models are analyzed.

1. INTRODUCTION

Distributed lags are a common tool in modeling dynamic processes, where a response to a certain stimulus is spread over time, and are frequently used in technical sciences as well as in economics. In economics, distributed lag models were proposed by Fisher (1937), Koyck (1954) and further developed by many others. In most cases these models are based on the assumption of a constant lag distribution. Such an approach is relevant whenever the modeled process is not sufficiently recognized and/or the alternative hypothesis has not been justified on the basis of available data. However, more and more instances of distributed lag models with the time-varying lag distribution can be met, Pesando (1972), Trivedi, Lee (1981), Otto (1985), Dahl, Kulaksizoglu (2004), Gadomski (1986, 2011), and others. An acquaintanceship with the mechanism causing significant changes in the lag distribution, which cannot be attributed solely to the impact of the random component, is a necessary condition for using distributed lag models with the time-varying lag distribution. While the theory of the distributed lag models with constant lag distributions has been developed, as for example Dhrymes (1981), Griliches (1967), Jorgensen (1966) (in econometrics, the emphasis is on the problem of estimation of such models), properties of the time-varying distributed lags have not been analyzed in any comparable extent.

A specific subclass of systems analyzed with the use of the distributed lag models are those where one can distinguish interconnected reservoirs/stocks and the inflows and outflows respectively feeding and exhausting, as well as connecting those stocks. For the purpose of this paper such systems will be called the systems of flows. Of particular interest are these systems where the outflows from stocks can be described by the distributed lag models. This paper is focused on the analysis of the distributed lag models with the time-varying lag distributions applied in modeling the systems of flows. Examples of such systems can be found in economics (stocks of fixed assets, inflows of investment, outflows of decommissioned assets; inventories of raw materials fed by replenishment, inflow and exhausted by consumption, etc.), technical sciences, demography, hydrology etc. Admitting the importance of the stochastic aspect of the distribution lag models, this paper is primarily focused on their deterministic properties.

In Section 2, basic terms and properties are introduced. In Section 3, a problem of duration of lagged response is analyzed in terms of the mean age of outflowing units and the mean value of the distributed lag. Section 4 contains an analysis of complex lag structures composed of parallelly or/and serially linked distributed lag models. Section 5 presents final conclusions.

Some variables and coefficients used in the paper have one or two subscripts; in order to avoid ambiguity, a separating coma is applied whenever an expression within the subscript could cause confusion.

2. BASIC PROPERTIES

Basic element of the flow system consists of a stock Z being fed by the inflow X and exhausted by the outflow Y. This property is described by the following stock equation:

$$z_t = z_{t-1} + x_t - y_t \quad (1)$$

where z_t stands for the level of stock at the end of period t, x_t and y_t, are respectively inflow and outflow rates in period t.

For the clarity of presentation it is assumed that there are no external gains or losses in the basic element, meaning that the system is leak-proof. This assumption can be relaxed whenever leaks can be presented in the form of an outflow described by the distributed lag model. Hence, the outflow in the basic element is given by the following relationship:

$$y_t = \sum_{i=0}^{\infty} \mathrm{v}_{t\,i}\ \mathrm{x}_{t\text{-}i} \ + \varepsilon_t \tag{2}$$

where:

x_t – value of an independent variable representing inflow X in period t,
y_t – value of an dependent variable representing outflow Y in period t,
$v_{t\,i}$ – coefficients of the lag structure $V_t=\{v_{t0}, v_{t1}, ... \}$ of the outflow given for the time period t; i = 0, 1, 2,…; fulfill the following conditions:

$0 \le v_{t\,i} \le 1$, i = 0, 1, 2,...

$a_{t=} \sum_{i=0}^{\infty} v_{t\,i} \le \infty$

ε_t – random component; independently distributed with zero mean and constant finite variance; this term will be omitted in further analysis.

A positive and finite coefficient a_t is called the long-term multiplier. Equation (2) can be rewritten in the following form:

$$y_t = a_t \sum_{i=0}^{\infty} w_{t\,i}\ x_{t-i} \ + \varepsilon_t \tag{2}$$

where coefficients w_{ti}, i = 0, 1, 2,…; are obtained by normalizing the coefficients of the lag structure V_t:

$$w_{t\,i} = \frac{v_{ti}}{\sum_{i=0}^{\infty} v_{t\,i}} = \frac{v_{ti}}{a_t},\ i = 0, 1, 2,\ldots \tag{3}$$

Coefficients w_{ti}, i = 0, 1, 2,...; constitute the lag distribution W_t, which has finite mean value $m(W_t)$ and variance $\sigma^2(W_t)$.

The independent variable X assumes non-negative values (one-way flow) and the dependent variable is obviously also non-negative. The latter is interpreted in the following way. Outflow y_t in the period t consists of: v_{t0} part of inflow x_t from the same period t, v_{t1} part of inflow x_{t-1} from the period t-1, v_{t2} part of inflow x_{t-2} from the period t-2, etc. In order to ascertain the level of stock to be non-negative, $z_t \geq 0$, or $y_t \leq z_{t-1} + x_t$, the following assumption concerning the inter-period dependence between the coefficients is adopted:

$$x_t \geq v_{t,0}\, x_t + v_{t+1,1}\, x_t + v_{t+2,2}\, x_t + \ldots = x_t \sum_{i=0}^{\infty} v_{t+i,i}$$

or

$$1 \geq v_{t,0,1} + v_{t+1,1} + v_{t+2,2} + \ldots = \sum_{i=0}^{\infty} \mathrm{v}_{\mathrm{t}+i,i}$$

which states, respecting the principle of mass conservation, that the sum of parts of the inflow x_t in the period t leaving the stock in the periods t, t+1, t+2,…; cannot exceed the amount x_t of inflow from the period t. Note that the case of strict inequality in the above constraint determines that some residual part of the inflow x_t remains in the stock "forever". A special case:

$$v_{t,0} + v_{t+1,1} + v_{t+2,2} + \ldots = \sum_{i=0}^{\infty} v_{t+i,i} = 1$$

determines that whatever flows into the stock must leave it within the infinite (or finite) time-horizon and has meaningful consequences for the distributed lag models with constant lag coefficients in which the sum of these coefficients is equal to 1.

Relationship (2) shows that the value of the dependent variable is determined by three following factors: (i) the present and past values of the independent variable, (ii) random variable ε_t and (iii) mechanism forming the lag structure of the outflow. As the interest of this paper is focused on the deterministic part of the analyzed problem, the random component ε_t will be dropped for the sake of clarity of presentation.

Variability of the lag structure can be caused by many factors, such as, for example, random, cyclical, having definite tendency. At this stage no additional assumptions concerning this variability are needed.

Quite often in the analysis of the systems of flows there emerges a problem of measuring time spent in the stock by units from the inflow. This issue is addressed later, however for now it will be assumed that the units which entered (flowed into) the stock in the current period are of the age 0, those from the preceding period of the age 1, from two periods back of the age 2, etc. Basing on this convention one can interpret the dependent variable as a mixture of the age-nonhomogeneous flows consisting of ($v_{t0}\, x_t$) part of inflow from the same period

x_t being 0 period old, ($v_{t1}\, x_{t-1}$) part of inflow from the previous period x_{t-1} being 1 period old, etc.

Solving equations (1) by recursive substitution of equation (2) k-1 times, k = 1, 2, 3,...; results in the following relationship:

$$z_t = z_{t-k} + \sum_{i=0}^{k-1} (x_{t-i} - y_{t-i})$$

$$z_t = z_{t-k} + \sum_{i=0}^{k-1} (1 - \sum_{j=0}^{i} v_{t-i+j,j}) x_{t-i} - \sum_{i=k}^{\infty} \sum_{j=0}^{k-1} v_{t-i+j,j} x_{t-i} \tag{4}$$

because for all j, j = 1, 2, 3,..; one gets

$$y_{t-j} = \sum_{i=0}^{\infty} v_{t-j,i}\; x_{t-j-i}$$

and

$$x_{t-j} - y_{t-j} = (1 - v_{t-j,0}) x_{t-j-i} - v_{t-j,1} x_{t-j-1} - v_{t-j,2}\, x_{t-j-2} - \cdots.$$

Adopting assumptions (based on a non-nonsense condition that $x_{t-i} \to 0$ with $i \to \infty$):

$$\lim_{k \to \infty} \sum_{i=k}^{\infty} \sum_{j=0}^{k-1} v_{t-i+j,j} x_{t-i} = 0$$

and $z_{-\infty} = 0$, one arrives at the following expression:

$$z_t = \sum_{i=0}^{\infty} (1 - \sum_{j=0}^{i} v_{t-i+j,\, j}) x_{t-i} \tag{5}$$

Motivation for the latter assumptions is based on the following elements:

$1 - v_{t0,0}$ is a part of inflow x_t from the period t, remaining in the stock Z at the end of period t,

$1 - v_{t-1,\, 0} - v_{t,\, 1}$, is a part of inflow x_{t-1} from the period t–1, remaining in the stock Z at the end of period t;

$1 - v_{t-2,0} - v_{t-1,1} - v_{t,2}$, is a part of inflow x_{t-2} from the period t–2, remaining in the stock Z at the end of period t; etc.

Having in mind the assumption concerning non-negative values of the stock z_t the values of the terms from equation (5)

$$1 - \sum_{j=0}^{i} v_{t-i+j,\,j}, \quad i = 0,1,2,\ldots;$$

are also non-negative, because, respecting the principle of mass conservation, at each period the following condition holds true:

$$v_{t+1,\,i+1} \leq 1 - \sum_{j=0}^{i} v_{t-i+j,\,j}, \quad i = 0,1,2,\ldots;$$

which states that in the period t+1 the part of the outflow y_{t+1} derived from the inflow x_{t-i} from the period t–i cannot exceed the part of the inflow x_{t-i} from the period t–i remaining in the stock at the end of period t (or at the beginning of the period t+1).

Consequently, the stock level z_t at the end of the period t (5) can be expressed in the alternative ways:

$$z_t = \sum_{i=0}^{\infty} s_{t,\,i} \mathrm{x}_{\mathrm{t}-i} \tag{6}$$

or

$$z_t = \sum_{i=0}^{\infty} z_{t\,i}, \tag{7}$$

where z_{ti}, i = 0, 1, 2, 3,…; denote the amount of inflow from the period t–i remaining within the stock at the end of period t.

Equation (5) shows that the level of stock at the end of period t can be also expressed as a distributed lag model with lag coefficients s_{ti}, i = 0, 1, 2, 3,…;

$$s_{t\,i} = 1 - \sum_{j=0}^{i} v_{t-i+j,\,j}\ ,\ i = 0,\ 1,\ 2,\ \ldots\ ; \tag{8}$$

note also that

$$s_{t+1,i+1} = s_{t,i} - v_{t+1,i+1};\ i = 0,\ 1,\ 2,\ 3,\ldots\ ;$$

which means that regardless of the shape of the lag structure V_t of the outflow, coefficients s_{ti}, $i = 0, 1, 2, 3,\ldots$; of the stock lag structure (5) form a monotonously diminishing sequence. In the case of the stock level, the impact of history is always a non-increasing function of the age of units residing within the stock.

Equation (6) reveals the time-structure of the stock, namely each variable $z_{t\,i}$, $i= 0, 1, 2, 3,\ldots$; represents amount of inflow x_{t-i} from the period $t-i$ remaining within the stock at the end of period t. This interpretation is used further in Section 3.

Consequences of equation (5) can be explained using as a reference the distributed lag model with constant lag distribution and the long term multiplier equal to 1. Assuming constant lag structure V of the outflow (the time index t is dropped), equation (6) can be rewritten in a simpler form (having in mind that in this case coefficients are normalized $w_i = v_i$, $i = 0, 1, 2, 3,\ldots$):

$$z_t = \sum_{i=0}^{\infty}(1 - \sum_{j=0}^{i} w_j)x_{t-i} \tag{8}$$

An important property of (8) is that it can also be expressed in the form:

$$z_t = m(W)\sum_{i=0}^{\infty} q_i x_{t-i} \tag{9}$$

where $m(W)$ is the mean value of the lag distribution W of the inflow and the coefficients q_i are defined by the following formula:

$$q_i = \frac{1 - \sum_{j=0}^{i} w_j}{m(W)}, \; i = 0, 1, 2, \ldots \tag{10}$$

constituting lag distribution Q of the stock because coefficients q_i, $i = 0, 1, 2, \ldots$; satisfy normalization conditions:

$$q_i \geq 0, \; i = 0, 1, 2, \ldots$$

$$\sum_{i=0}^{\infty} q_i = 1 \,.$$

Proving (9) is equivalent to proving that

$$\sum_{i=0}^{\infty} s_i = \sum_{i=0}^{\infty}(1-\sum_{j=0}^{i} w_j) = m(W).$$

Proof. On the basis of (5) successive coefficients v_i, $i= 0, 1, 2, \ldots$; can be expressed by the following equations:

$$\begin{aligned}
s_0 &= 1-w_0 &&= w_1+w_2+w_3+w_4+\ldots\\
s_1 &= 1-w_0-w_1 &&= w_2+w_3+w_4+\ldots\\
s_2 &= 1-w_0-w_1-w_2 &&= w_3+w_4+\ldots\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}$$

Summing coefficients s_i, $i = 0, 1, 2, \ldots$; is equivalent to summing the right-hand sides of the above equations, which results in the sum of the following terms: one w_1, two w_2, three w_3, etc, or:

$$\sum_{i=0}^{\infty} i \cdot w_i = m(W),$$

which was to be shown.

Equation (9) is important as it shows that in the steady state, when $x_t = x^*$ for all t, the following relation holds true:

$$z_t = m(W)\, x^*.$$

Returning to the time-varying lag coefficients we can rewrite equation (5) in the following form:

$$z_t = b_t \sum_{i=0}^{\infty} q_{t\,i} x_{t-i} \tag{11}$$

where

$$q_{t,i} = \frac{1-\sum_{j=0}^{i} v_{t-i+j,j}}{\sum_{i=0}^{\infty}\left(1-\sum_{j=0}^{i} v_{t-i+j,j}\right)},\ q_{t,i} \geq 0,\ i = 0, 1, 2, \ldots; \tag{12}$$

$$\sum_{i=0}^{\infty} q_{t\,i} = 1$$

$$b_t = \sum_{i=0}^{\infty}\left(1 - \sum_{j=0}^{i} v_{t-i+j,\,j}\right).$$

Due to the above properties it should be noted that the coefficients $q_{t\,i}$, $q_{ti} \geq 0$, $i = 0, 1, 2, \ldots$; of the distributed lag model (11) are functions of the past lag structures V_t, V_{t-1}, $V_{t-2}, \ldots$; and constitute lag distribution Q_t of the stock.

An analysis of a relationship between the outflow and stock lag distributions will be limited to the models with the time-constant lag distribution.

Mean value of the lag distribution of stock Q in model (10) is given by the following formula:

$$m(Q) = \sum_{i=1}^{\infty} i s_i = \frac{1}{2}\left[\frac{\sigma^2(W)}{m(W)} + m(W) - 1\right]. \tag{13}$$

Proof of the above is as follows. On the basis of the definition we have:

$$m(Q) = \sum_{i=1}^{\infty} i\, q_i = \sum_{i=1}^{\infty} i \frac{1 - \sum_{j=0}^{i} w_j}{m(W)} =$$

$$= \frac{1}{m(W)}[1 \cdot (1 - w_0 - w_1) +$$

$$+ 2 \cdot (1 - w_0 - w_1 - w_2) +$$

$$+ 3 \cdot (1 - w_0 - w_1 - w_2 + w_3) +$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots].$$

Taking into account that

$$1 - \sum_{j=0}^{i} w_j = \sum_{j=i+1}^{\infty} w_j$$

the above equation can be rewritten in the following form:

$$\sum_{i=1}^{\infty} i \frac{1-\sum_{j=0}^{i} w_j}{m(W)} = \frac{1}{m(W)}[1 \cdot (w_2 + w_3 + w_4 + \ldots\ldots\ldots\ldots\ldots\ldots) +$$

$$+ 2 \cdot (\qquad w_3 + w_4 + \ldots\ldots\ldots\ldots\ldots\ldots) +$$

$$+ 3 \cdot (\qquad\qquad w_4 + \ldots\ldots\ldots\ldots\ldots\ldots) +$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots] =$$

$$= \frac{1}{m(W)} \sum_{i=2}^{\infty} \left(\sum_{j=1}^{i-1} j \right) w_i = \frac{1}{m(W)} \sum_{i=2}^{\infty} \frac{i(i-1)}{2} w_i = \frac{1}{2m(W)} \sum_{i=2}^{\infty} (i^2 w_i - i w_i).$$

Having in mind that:

$$\sum_{i=2}^{\infty} i^2 w_i = \sum_{i=1}^{\infty} i^2 w_i - w_1 \ ,$$

$$\sum_{i=2}^{\infty} i\, w_i = \sum_{i=1}^{\infty} i\, w_i - w_1$$

and

$$\sigma^2(W) = \sum_{i=2}^{\infty} i^2 w_i - m^2(W)$$

by rearranging terms one arrives at:

$$m(Q) = \frac{1}{2m(W)} \sum_{i=2}^{\infty} (i^2 w_i - i w_i) = \frac{1}{2m(W)} \left[\sum_{i=1}^{\infty} i^2 w_i - m(W) \right] =$$

$$= \frac{1}{2m(W)} \left[\sigma^2(W) + m^2(W) - m(W) \right] = \frac{1}{2} \left[\frac{\sigma^2(W)}{m(W)} + m(W) - 1 \right];$$

which was to be shown.

Note that in the case of the simplest one-point lag distribution with only τ-th non-zero coefficient $v_\tau \neq 0$ equation (13) may be reduced to the following one:

$$m(Q) = \frac{1}{2}(\tau - 1),$$

because the variance in this case equals zero.

The above considerations are illustrated by the following examples.

Example 1. Assume the following model:

$z_t = z_{t-1} + x_t - y_t,$

$y_t = \lambda(x_t + z_{t-1});$

which is equivalent to the distributed lag model of the outflow y_t with regard to the inflow x_t with a constant lag distribution of the outflow, $w_i=\lambda(1-\lambda)^i$, $i = 0, 1, 2, 3,..;$ (of the Koyck type), $\lambda\in(0,1)$ and the long-term multiplier equal to 1. This equivalence is easily proved by recursive substitution:

$y_t=\lambda(x_t+z_{t-1})=\lambda x_t+\lambda z_{t-1}=\lambda x_t+\lambda(1-\lambda)x_{t-1}+\lambda(1-\lambda)z_{t-2}=$

$$= \frac{\lambda}{1-\lambda}\sum_{i=0}^{n}(1-\lambda)^{i+1}x_{t-i} + \frac{\lambda}{1-\lambda} z_{t-1-n}\sum_{i=0}^{n}(1-\lambda)^{i+1}$$

and finally:

$$y = \frac{\lambda}{1-\lambda}\sum_{i=0}^{\infty}(1-\lambda)^{i+1}x_{t-i}.$$

Note that the second of the two above presented equations defining the model fulfills the mass conservation principle.

As a particular case this model is important and interesting because its lag distribution of the stock is identical to the lag distribution of the outflow.

Proof of the above property is based on expression:

$$s_i = 1 - \sum_{j=0}^{i} w_j = \frac{1-\lambda}{\lambda}\sum_{j=0}^{i}(1-\lambda)^{j+1} = (1-\lambda)^{i+1}.\ i = 0, 1, 2, 3,..\ .$$

Sum of coefficients of the lag structure of the stock s_i, $i = 0, 1, 2, 3,...;$ equals the mean value $m(W)$ of the lag distribution W:

$$\sum_{i=0}^{\infty} s_i = \sum_{i=0}^{\infty}(1-\lambda)^{i+1} = \frac{1-\lambda}{\lambda} = m(W)$$

so that:

$$q_i = \frac{1-\sum_{j=0}^{i} \mathrm{w}_j}{m(W)} = \frac{(1-\lambda)^{i+1}}{\frac{1-\lambda}{\lambda}} = \lambda(1-\lambda)^i = w_i \text{ , } i = 0, 1, 2, \dots \text{ .}$$

It should be emphasized that the equality of both the outflow and stock lag distributions illustrated in Example 1 is exceptional. In a common case one should expect different lag distributions of the outflow and stock. An example is presented below.

Example 2. Assume a uniform lag distribution of the outflow such that $w_i=1/(n+1)$ for $i = 0, 1, 2,..., n$; and $w_i = 0$ for $i > n$, where $n>0$ is a natural finite number and the mean value of the lag distribution of the outflow $m(W)= n/2$.

In this example, one gets uniformly decreasing coefficients of the linear lag structure of the stock $v_i=1-(i+1)/(n+1)=(n-i)/(n+1)$ for $i = 0, 1, 2,..., n;$ and $v_i=$ 0 for $i \geq n$; and the mean value of the lag distribution of the stock $m(Q)=(n-1)/3$. Therefore both distributions and respective mean values are unequal.

Example 3. Assume the following time-varying model:

$$z_t= z_{t-1} + x_t - y_t,$$

$$y_t= \lambda_t(x_t + z_{t-1});$$

which differs from the one presented in Example 1 in the variability of the coefficient λ_t, $\lambda_t \in (0,1)$.

Solving the above model by recursive substitution gives the following results:

– coefficients of the outflow lag distribution:

$$v_{ti} = \frac{\lambda_t}{1-\lambda_t} \prod_{j=0}^{i} \left(1-\lambda_{t-j}\right), \; i = 0, 1, 2,..$$

– coefficients of the stock lag distribution:

$$s_{ti} = \prod_{j=0}^{i} \left(1-\lambda_{\,t-j}\right), \; i = 0, 1, 2,.. .$$

In this example, it is noteworthy that although the principle of the mass conservation is respected (meeting for all t the condition: $0 \leq y_t \leq z_{t-1} + x_t$), the coefficients of the outflow lag structure do not necessarily sum to one.

3. MEASURES OF LAGS IN THE FLOW SYSTEMS

Measuring the scope of lags is straightforward only in the case of a simple one-point lag of a definite number of periods (single point lag distribution). In the more general case of the distributed lags, the commonly used measures of duration of the process of transferring impact of the change in the independent variable on the dependent variable are the mean value $m(W_t)$ and (seldom used) median $\eta(W_t)$ of the lag distribution. However, as it was shown in Gadomski (2011), these measures do not account for the dynamics of the independent variable. This drawback can be, to a certain extent, corrected by using the concept of the mean (and/or median) of the so-called resultant lag distribution, which was proposed in Gadomski (2011) in order to capture separate impacts of both the time-varying lag distribution and the behavior of the independent variable.

The shares $u_{t\,i}$, $i = 0, 1, 2,...$ and the mean $m(U_t)$ of the resultant lag distribution U_t of stock are defined by the following expressions:

$$u_{ti} = \frac{v_{t\,i} x_{t-i}}{y_t}, \; i = 0, 1, 2,... ; \tag{14}$$

$$m(U_t) = \sum_{i=1}^{\infty} i \cdot u_{t\,i} \tag{15}$$

The coefficients u_{ti}, $i = 0, 1, 2,...$; equation (14), which constitute the resultant lag distribution of the outflow U_t are interpreted as shares of the units of inflow from the period $t–i$, $i = 0, 1, 2,...$; in the outflow in period t. The mean of the resultant lag distribution $m(U_t)$ of the outflow, equation (15), is the average age of units of the outflow in the period t.

Note that in the context of the systems of flows the mean value $m(W_t)$ of the lag distribution W_t of the outflow can be interpreted as the mean time the unit of outflow spent in the stock being in the steady state, or as the mean age of units flowing out of stock in the steady state. Whenever the system is beyond the steady state, these interpretations are not valid, however the mean $m(U_t)$ of the resultant lag distribution U_t of the outflow retains this interpretation. Of course, in the steady state both measures, the mean value $m(W_t)$ of the lag distribution W_t and the mean $m(U_t)$ of the resultant lag distribution U_t, are equal.

A similar path of reasoning applied to equation (10) enables an analysis of the mean age of units residing within the stock. In this context one can interpret the mean value $m(Q_t)$ of the lag distribution Q_t of the stock as the average age of units residing in the stock in the steady state, beyond which the adequate measure is the mean $m(P_t)$ of the resultant lag distribution of stock P_t defined as follows:

$$p_{t\,i} = \frac{s_{t\,i} x_{t-i}}{z_t} = \frac{\sum_{i=0}^{\infty}\left(1-\sum_{j=0}^{i} v_{t\,i}\right)x_{t-i}}{z_t} = \frac{z_{t\,i}}{z_t},\ i = 0, 1, 2,... \tag{16}$$

$$m(P_t) = \sum_{i=1}^{\infty} i \cdot p_{t\,i} = \sum_{i=1}^{\infty} i \cdot \frac{z_{t\,i}}{z_t} \tag{17}$$

4. COMPLEX STRUCTURES OF THE DISTRIBUTED LAG MODELS

An important issue concerning the properties of the distributed lags is the one related to their structure. In this paper, complex distributed lag models are those models which are the result of a sum and/or superposition of the distributed lag models.

The sum of n distributed lag models is defined in the following way:

$$y_t = y_t^{(1)} + y_t^{(2)} + ... + y_t^{(n)}; \quad n < \infty \tag{18}$$

where:

n – number of summed models, $n < \infty$

$y_t^{(j)}$ – dependent variables, $j= 1, 2,..,n;$ having the form:

$$y_t^{(1)} = \sum_{i=0}^{\infty} v_{t\,i}^{(1)} x_{t-i} + \varepsilon_t^{(1)},\ y_t^{(2)} = \sum_{i=0}^{\infty} v_{t\,i}^{(2)} x_{t-i} + \varepsilon_t^{(2)}, \cdots,\ y_t^{(n)} = \sum_{i=0}^{\infty} v_{t\,i}^{(n)} x_{t-i} + \varepsilon_t^{(n)},$$

$\varepsilon_t^{(j)}$ – independently distributed random component; $j=$ 1, 2,.., $n;$ with zero mean and constant finite variance.

It can be easily shown that the sum (18) is also a distributed lag model with regard to the independent variable x_t :

$$y_t = \sum_{i=0}^{\infty} v_{t\,i} x_{t-i} + \varepsilon_t = a_t \sum_{i=0}^{\infty} w_{t\,i} x_{t-i} + \varepsilon_t;$$

where:

$$v_{ti} = \sum_{j=1}^{n} v_{ti}^{(j)};\ a_t^{(j)} = \sum_{i=0}^{\infty} v_{ti}^{(j)};\ a_t = \sum_{j=1}^{n} a_t^{(j)} = \sum_{j=1}^{n}\sum_{i=0}^{\infty} v_{ti}^{(j)};\ w_{ti} = \sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} w_{ti}^{(j)};$$

$$\varepsilon_t = \sum_{j=1}^{n} \varepsilon_i^{(j)}.$$

If each of the n summed distributed lag models has lag distribution $W_t^{(j)}$ and finite mean $m(W_t^{(j)})$, $j = 1, 2, ..., n$, then the mean $m(W_t)$ of the lag distribution W_t is the following:

$$m(W_t) = \frac{a_t^{(1)}}{a_t} m(W_t^{(1)}) + \frac{a_t^{(2)}}{a_t} m(W_t^{(2)}) + \ldots + \frac{a_t^{(n)}}{a_t} m(W_t^{(n)}) . \tag{19}$$

The above relationship is obtained from the definition:

$$m(W_t) = \frac{\sum_{i=1}^{\infty} i \sum_{j=1}^{n} v_{ti}^{(j)}}{\sum_{i=0}^{\infty}\sum_{j=1}^{n} v_{ti}^{(j)}} = \frac{\sum_{j=1}^{n}\sum_{i=1}^{\infty} i a_t^{(j)} w_{ti}^{(j)}}{\sum_{j=1}^{n}\sum_{i=0}^{\infty} v_{ti}^{(j)}} = \frac{\sum_{j=1}^{n} a_t^{(j)} \sum_{i=1}^{\infty} i w_{ti}^{(j)}}{\sum_{j=1}^{n} a_t^{(j)}} = \sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} m(W_t^{(j)}) .$$

The variance of the lag distribution of the sum of distributed lag models is given by the following formula:

$$\sigma^2(W_t) = \sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} \sigma^2(W_t^{(j)}) + \left\{ \sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} m(W_t^{(j)}) - \left[\sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} m^2(W_t^{(j)}) \right]^2 \right\} . \tag{20}$$

Proof of the relationship (20) makes use of the formula:

$$\sum_{i=0}^{\infty} i^2 \sum_{j=0}^{n} \frac{a_t^{(j)}}{a_t} w_{ti}^{(j)} = \sum_{j=0}^{n} \frac{a_t^{(j)}}{a_t} \sum_{i=0}^{\infty} i^2 w_{ti}^{(j)} = \sum_{j=1}^{n} \frac{a_t^{(j)}}{a_t} \left[\sigma^2(W_t^{(j)}) + m^2(W_t^{(j)}) \right]$$

On this basis one can write:

$$\sigma^2(W_t)=\sum_{j=1}^{n}\frac{a_t^{(j)}}{a_t}\left[\sigma^2(W_t^{(j)})+m^2(W_t^{(j)})\right]-\left[\sum_{j=1}^{n}\frac{a_t^{(j)}}{a_t}m(W_t^{(j)})\right]^2$$
$$=\sum_{j=1}^{n}\frac{a_t^{(j)}}{a_t}\sigma^2(W_t^{(j)})+\left\{\sum_{j=1}^{n}\frac{a_t^{(j)}}{a_t}m^2(W_t^{(j)})-\left[\sum_{j=1}^{n}\frac{a_t^{(j)}}{a_t}m(W_t^{(j)})\right]^2\right\}$$

Note that on the basis of the Cauchy-Schwarz inequality, the difference within the curly bracket is always non-negative so that the variance of the sum is greater than the weighted mean of individual variances. However, in the case when all lag distributions have the same mean value, the variance of the sum of lag distributions equals the weighted sum of individual variances.

Note that the random component ε_t is a simple sum of random components of individual models. In the case of the superposition of the distributed lag models, this is much more complicated. As our interest is focused on the deterministic part, in further analysis the random component will be omitted, however this aspect is worth considering.

Note also that any distributed lag model having a general form of (2) is in fact the sum of simple lags of the form:

$$y_t^{(1)}=v_{t0}x_t+\varepsilon_t^{(1)},\ y_t^{(2)}=v_{t1}x_{t-1}+\varepsilon_t^{(2)},\cdots.$$

Superposition of the distributed lag models occurs whenever the independent variable in the distributed lag model is the dependent variable of another distributed lag model. Assume (for simplicity that random components are dropped)

$$y_t=\sum_{i=0}^{\infty}v_{t\,i}^{(n)}x_{t-i}^{(n)},\ x_t^{(n)}=\sum_{i=0}^{\infty}v_{t\,i}^{(n-1)}x_{t-i}^{(n-1)},\ldots,x_t^{(2)}=\sum_{i=0}^{\infty}v_{t\,i}^{(1)}x_{t-i}^{(1)}. \tag{21}$$

In order to facilitate the manipulation of the formula, the above models can be presented in the form of the polynomial operators:

$$y_t=V_t^{(n)}(L)x_t^{(n)},\ x_t^{(n)}=V_t^{(n-1)}(L)x_t^{(n-1)},\ldots,x_t^{(2)}=V_t^{(1)}(L)x_t^{(1)}, \tag{22}$$

where

L – shift operator having the following properties:

$L\,x_t=x_{t-1}$

$L^2\,x_t=L\,L\,x_t=L\,x_{t-1}=x_{t-2}$

$L^k x_t = x_{t-k}$; $k = 0, +/-1, +/-2,\ldots$;
$L^k L^l = L^{k+l}$; $k, l = 0, +/-1, +/-2,\ldots$;
$L^0 x_t = 1\, x_t = x_t$
$L^k L^{-k} = L^{k-k} = L^0 = 1$; $k = 0, +/-1, +/-2,\ldots$;
$(c_1 L^k + c_2 L^l)\, x_t = c_1 x_{t-k} + c_2 x_{t-l}$; $k, l = 0, +/-1, +/-2,\ldots$;
$V_t^{(j-1)}(L)$ – polynomial of the form:

$$V_t^{(j+1)}(L) = \sum_{i=0}^{\infty} v_{ti}^{(j)} L^i .$$

Assuming that each structure $V_t^{(j)}$; $j = 1, 2, \ldots$ has the lag distribution and respective mean value of the lag distribution, a consecutive substitution of terms in (22) results in the following expression:

$$\begin{aligned} y_t &= V_t(L)x_t^{(1)} = V_t^{(n)}(L)V_t^{(n-1)}(L)\cdot\ldots\cdot V_t^{(1)}(L)x_t^{(1)} \\ &= a_t^{(1)}W_t^{(n)}(L)a_t^{(n-1)}W_t^{(n-1)}(L)\cdot\ldots\cdot a_t^{(1)}W_t^{(1)}(L)x_t^{(1)} \\ &= \prod_{i=1}^{n} a_t^{(i)} \prod_{i=1}^{n} W_t^{(i)}(L)x_t^{(1)} = a_t V_t(L)x_t^{(1)} \end{aligned} \tag{23}$$

where

$$V_t(L) = V_t^{(n)}(L)V_t^{(n-1)}(L)\cdot\ldots\cdot V_t^{(1)}(L) = \prod_{i=1}^{n} V_t^{(i)}(L)$$

is the polynomial operator and a_t is the long-term multiplier of the superposition of the distributed lag models:

$$a_t = \prod_{i=1}^{n} a_t^{(i)} .$$

In the following analysis we will use the generating function built on the coefficients of the lag distribution having the following form:

$$W_t^{(j)}(\theta) = \sum_{i=0}^{\infty} w_{ti}^{(j)} \theta^i ,\ j = 1, 2, \ldots .$$

The following properties of this function will be used:

$$W_t^{(j)}(1) = 1$$

$$\frac{dW_t^{(j)}(1)}{d\theta} = m(W_t^{(j)})$$

$$\sigma^2(W_t^{(j)}) = \frac{d^2W_t^{(j)}(1)}{d\theta^2} + \frac{dW_t^{(j)}(1)}{d\theta} - \left[\frac{dW_t^{(j)}(1)}{d\theta}\right]^2$$

Note that W_t is also a lag distribution; its coefficients are non-negative and they sum to 1, as a consequence of the fact that the generating function built on the lag distribution W_t, $W_t(\theta)$ in the point $\theta = 1$ equals 1, because for each

$$W_t^{(j)}(1) = 1 \text{ , } j = 1, 2, \ldots ;$$

so that:

$$W_t(1) = \prod_{j=1}^{n} W_t^{(j)}(1) = 1 \, .$$

Mean value $m(W_t)$ of the lag distribution W_t is described by the following expression:

$$m(W_t) = m(W_t^{(1)}) + m(W_t^{(2)}) + \ldots + m(W_t^{(n)}) \qquad (24)$$

Proof of equation (24) is based on the formula relating the first derivative of the generating function in point $\theta = 1$ and the differential of the product:

$$\frac{dW_t(1)}{d\theta} = \frac{d\left[W_t^{(1)}(\theta)\cdot\ldots\cdot W_t^{(n)}(\theta)\right]}{d\theta}_{|\theta=1}$$

$$= \left\{\prod_{i=1}^{n} W_t^{(i)}(\theta)\left[\sum_{i=1}^{n}\frac{dW_t^{(i)}(\theta)}{d\theta}\frac{1}{W_t^{(i)}(\theta)}\right]\right\}_{|\theta=1}$$

$$= \sum_{i=1}^{n}\frac{dW_t^{(i)}(1)}{d\theta} = \sum_{i=1}^{n} m(W_t^{(i)}) = m(W_t).$$

Variance $\sigma^2(W_t)$ of the lag distribution W_t is the sum of the variances of the component lag distributions $\sigma^2(W_t^{(i)})$, $i = 1, 2, \ldots$:

$$\sigma^2(W_t) = \sigma^2(W_t^{(1)}) + \sigma^2(W_t^{(2)}) + \ldots + \sigma^2(W_t^{(n)}) \tag{25}$$

Proof. Because we have

$$\frac{d^2 W_t(\theta)}{d\theta^2} =$$

$$= \prod_{i=1}^{n} W_t^{(i)}(\theta) \left\{ \sum_{i=1}^{n} \frac{d^2 W_t^{(i)}(\theta)}{d\theta^2} \frac{1}{W_t^{(i)}(\theta)} + \left[\sum_{i=1}^{n} \frac{dW_t^{(i)}(\theta)}{d\theta} \frac{1}{W_t^{(i)}(\theta)} \right]^2 - \sum_{i=1}^{n} \left[\frac{dW_t^{(i)}(\theta)}{d\theta} \frac{1}{W_t^{(i)}(\theta)} \right]^2 \right\}$$

$$\frac{d^2 W_t(1)}{d\theta^2} = \sum_{i=1}^{n} \frac{d^2 W_t^{(i)}(1)}{d\theta^2} + \left[\sum_{i=1}^{n} m\left(W_t^{(i)}\right) \right]^2 - \sum_{i=1}^{n} m^2\left(W_t^{(i)}\right).$$

Accounting for the above leads to the following:

$$\sigma^2(W_t) = \sum_{i=1}^{n} \frac{d^2 W_t^{(i)}(1)}{d\theta^2} + \sum_{i=1}^{n} m\left(W_t^{(i)}\right) - \sum_{i=1}^{n} m^2\left(W_t^{(i)}\right)$$

$$= \sum_{i=1}^{n} \left[\frac{d^2 W_t^{(i)}(1)}{d\theta^2} + m\left(W_t^{(i)}\right) - m^2\left(W_t^{(i)}\right) \right] = \sum_{i=1}^{n} \sigma^2(W_t^{(i)})$$

which was to be shown.

5. FINAL REMARKS

Distributed lag models of the flow systems are the subclass of the distributed lag models and are often applied in modeling. The variability of the lag structure makes them versatile. These models are characterized by a certain capacity; this is a stock of what entered the process of delay and has not reappeared in the outflow. It was shown that the amount in that stock also has the properties of the distributed lag with the parameters of the lag distribution derived in explicit formulae.

The performed analysis showed the relevance of the concept of the so called resultant distribution and parameters of the resultant distribution: mean and variance. The latter parameters in the context of the distributed lag models of flows have useful and meaningful interpretation.

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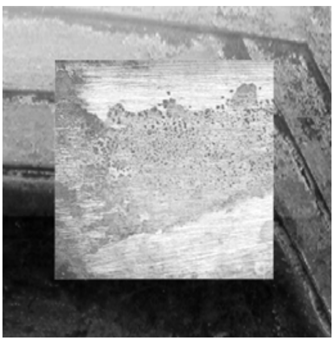
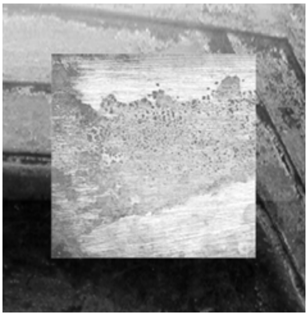


ABSTRACTS

MACROMODELS 2011
Łódź 2012, p. 75



Łukasz T. Gątarek, Koen Hooning,
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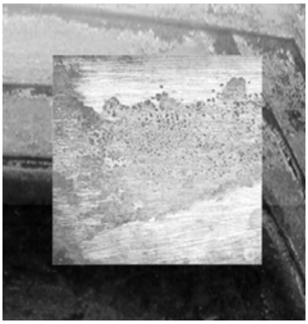
Censoring in Metropolis -Hastings Sampling Algorithm. Left-Tail Prediction with GARCH models

Abstract. The etension of the Metropolis-Hastings algorithm is proposed by replacing the standard likelihood with the censored likelihood for the acceptance-rejection weights evaluation. Further, the Bayesian Model Averaging is extended by replacing the predictive likelihood by the censored predictive likelihood. The use of censored Metropolis-Hastings and censored predictive likelihood offers a focus on the left tail of the distirbution. We perform an extensive simulation study to investigate the ability of these methods to outperform the standard sampling algorithm and traditional Bayesian Model Averaging techniques with application to the GARCH models and Value-at-Risk methods (coverage probability).



MACROMODELS 2011
Łódź 2012, p. 76

Jacek Osiewalski, Krzysztof Osiewalski
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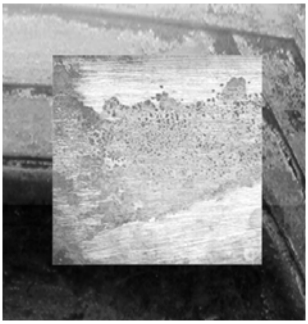
General Hybrid MSV–MGARCH Models of Multivariate Volatility – Bayesian Approach

Abstract. J. Osiewalski and A. Pajor considered a new class of volatility specifications, the hybrid multivariate stochastic variance – GARCH (MSV–MGARCH) models, where the conditional covariance matrix is a product of a univariate latent process and a matrix with a simple MGARCH structure (Engle's DCC, scalar BEKK). The aim was to parsimoniously describe volatility of a large group of assets. Their hybrid models, similarly as other specifications from the MSV class, require the Bayesian approach equipped with efficient MCMC simulation tools. The numerical effort should pay – the hybrid MSF–SBEKK(1,1) type I model seems very useful due to its good fit and ability to jointly cope with as many as 50 assets. However, one latent process may be insufficient in the case of a highly heterogenous portfolio. In this study we propose a general hybrid MSV–MGARCH structure that uses as many latent processes as there are relatively homogenous groups of assets. We present full Bayesian inference for such models and suggest MCMC simulation strategy. The proposed approach is used to jointly model volatility on very different markets. We formally compare joint modelling to individual modelling of volatility on each market.

MACROMODELS 2011
Łódź 2012, p. 77



Krzysztof Osiewalski, Jacek Osiewalski
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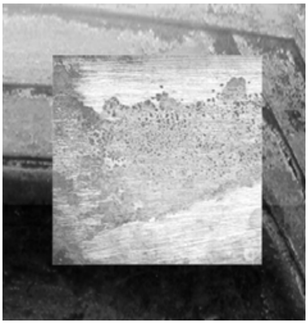
Missing Observations in Volatility Contagion Analysis. Bayesian Approach Using the MSV-MGARCH Framework

Abstract. Usually observations of prices on different markets are not fully synchronous. The question is whether we should exclude from modelling the days with prices not available on all markets (thus losing some information and implicitly modifying the time axis) or somehow complete the missing (non-existing) prices. In order to compare the effects of each of two ways of dealing with partly available data, one should consider formal procedures of replacing the unavailable prices by their appropriate predictions. We propose a fully Bayesian approach, which amounts to obtaining the marginal predictive distribution for any particular day in question. This procedure takes into account uncertainty and can be used to check validity of informal ways of "completing" the data. We use simple hybrid MSV-MGARCH models, which can parsimoniously describe volatility of a large number of prices or indices. In order to conduct Bayesian inference, the conditional posterior distributions for all unknown quantities are derived and the Gibbs sampler (with Metropolis-Hastings steps) is designed. Our approach is applied to daily prices from different financial and commodity markets, including the period of the global financial crisis. We compare inferences about conditional correlation obtained in the cases of deleted or completed observations.



MACROMODELS 2011
Łódź 2012, p. 78

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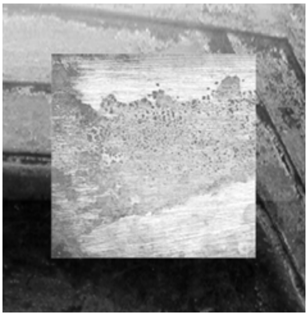
A Bayesian Analysis of Exogeneity in Models with Latent Variables

Abstract. The paper presents some results on exogeneity in models with latent variables. The concept of exogeneity is extended to the class of models with latent variables, in which a subset of parameters and latent variables is of interest. Exogeneity is discussed from the Bayesian point of view. We propose sufficient weak and strong exogeneity conditions in the vector error correction model (VECM) with stochastic volatility (SV) disturbances. Finally, an empirical illustration based on the VECM-SV model for the daily growth rates of two main official Polish exchange rates: PLN/USD, PLN/EUR and USD/EUR from the international Forex market is presented. The exogeneity of the USD/EUR rate is examined. The strong exogeneity hypothesis of the USD/EUR rate is not rejected by the data.

Keywords: Exogeneity; Bayesian cuts; Latent variables; Noncausality; Stochastic volatility

JEL classification: C11, C32.



MACROMODELS 2011
Łódź 2012, p. 79

Justyna Wróblewska
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Bayesian Analysis of Common Cyclical Features in VEC Models

Abstract. Once the concept of cointegration that enables the proper statistical analysis of long-run comovements between unit root processes has been introduced it became the subject of interest of many economic investigators. However, investigation of short-run comovement between economic time series seems as much important, especially for economic decision-makers. The concept of common features, introduced by Engle, Kozicki (1993), may be of some help. Based on this idea Hecq, Palm, Urbain (2006) have discussed two forms of the reduced rank structure in a VEC model: strong and weak. The strong form reduced rank structure (SF) takes place when at least one linear combination of the first diferences of the variables exists, which is a white noise. However, when this assumption seems too strong, the weaker case can be considered. The weak form appears when the linear combination of first diferences adjusted for long-run efects exists, which is a white noise. The main focus of this paper is Bayesian analysis of the VEC models involving these two forms of reduced rank restrictions. After the introduction and discussion of such Bayesian models, the presented methods will be illustrated via an empirical investigation of the price -wage spiral in the Polish economy.

MODELLING ECONOMIES
IN TRANSITION
2011

AMFET

Association for Modelling and Forecasting
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Modelling Economies in Transition 2011
Łódź 2012

CONTENTS

Modelling Economies in Transition 2011
Łódź 2012,

PREFACE

The papers published in this monograph represent the recent contributions to the economic modelling sponsored by the Association for Modelling and Forecasting Economies in Transition (AMFET). They were presented at the 16th AMFET Conference on Modelling Economies in Transition held jointly with the XXXVIII International Conference MACROMODELS 2011 in Poznań, Poland, November 30 - December 3, 2011. This conference was organised by the Association for Modelling and Forecasting Economies in Transition (AMFET) and the Department of Econometric Models and Forecasts of the Institute of Econometrics, University of Łódź.

The 16th AMFET Conference followed the tradition of international meetings held each year to discuss the issues of transition and market economies development. They apply both to the countries that became the members of the European Union and to the remaining East- and South-European countries. These discussions are based on studies exploring the modelling exercises. Their outcomes are published every year in AMFET Monographs and scientific journals.

The meetings offer the floor for discussions on the developments of models' specification from transition economies to market economies and their use, and on applied econometric methodology. They especially cover issues on the use of models for the European economies in forecasting and medium- and long-term simulation exercises. Recently the models are also applied to analyses related to impacts of monetary and fiscal policies. The meetings remain open for more general discussion on macroeconomic modelling of the accession and post-accession process to the EU.

The participants of these meetings are mainly representatives of the Central and East European countries. However, there is an increasing representation of the West European scholars.

The Conference took place in Poznań. It helped to provide a unique conference atmosphere. It was to a large extent the result of the efforts of its organisers led by Professor Aleksander Welfe and of the Secretary of the Programme Committee Michał Majsterek.

AMFET put emphasis on organising sessions devoted mainly to models of emerging markets, their use in simulation exercises, especially regarding the results of the EU accession.

The content of the AMFET Monograph is organised in the following way. The papers appear under the following topics:

- Consumption and Investment,
- Economic Growth and Business Cycle,
- Financial Crisis.

All articles were reviewed. The papers accepted for publication were revised by the authors prior to the regular editorial process. For the papers not submitted for publication in this monograph the summaries are provided. The English language and editorial verification has been performed by Waldemar Florczak.

On December 2, 2011, AMFET annual meeting took place. Professor Marek Gruszczyński, the President of the Association, presented the annual report for the year 2010. New Board was declared that consists of the following persons: President – Ryszard Doman, President-elect – Mateusz Pipień, Past-President – Marek Gruszczyński, Treasurer – Iwona Konarzewska, Secretary – Piotr Wdowiński, Member –Władysław Welfe, Member – Reinhard Neck, Member Mariusz Plich.

Last but not least, we would like to thank the Polish Academy of Science for financial support, and numerous colleagues, and editorial staff for their efforts, which made it possible to publish this monograph.

Władysław Welfe

Łódź, November 2012

CONSUMPTION AND INVESTMENT

Modelling Economies in Transition 2011
Łódź 2012

Modelling Economies in Transition 2011
Łódź 2012, pp. 89–107

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Business Investment Functions

Abstract. Business investment functions are an important component of macroeconometric models. In the W models of the Polish Economy the equations explain separately the investment in machinery and equipments and in buildings. They are based on the accelerator rule, allowing for the user costs of investment. The experience shows that the variability of investment rates of growth cannot be fully explained by relevant rates of growth of GDP only. Hence, an attempt was undertaken to introduce a proxy for the risk premium. Several indicators were tried. The best occurred the ratio of budget deficit to the GDP. The empirical results are shown in the paper.

INTRODUCTION

The demand for fixed capital of enterprises is mostly addressed to its global volume, treating it as homogenous. However, in many models it is disaggregated into the demand for machinery, equipment and for buildings and structures.

1. THE DEMAND FUNCTIONS FOR BUSINESS INVESTMENT

The demand for fixed capital is firstly confronted with its existing stock. The implementation of demand may first lead to an increase of the rate of utilization of fixed capital. It is, however, most frequently associated with the necessity to increase its stock due to new investments. This process can be approximated, assuming that there are adaptive adjustments leading to generation of the stock of fixed capital at the end of the period K_t.

Let us define:

$$K_t - K_{t-1} = \gamma\left(K_t^* - K_{t-1}\right), \tag{1}$$

Following the demand function for fixed capital obtained from solving for K_t^* from the Cobb-Douglas production function we can write:

$$K_t^* = CA_t^{-1/\alpha} X_t^{1/\alpha} N_t^{-\beta/\alpha} \left(R_t / P_t\right)^{-\nu/\alpha} e^{\varepsilon_t}, \tag{2}$$

where: C – constant, A_t – total factor productivity, N_t – employment, ε_t – random component, $\alpha > 0$ elasticity of production with respect to fixed capital, $\beta > 0$ elasticity of production with respect to employment, v elasticity with respect to real interest rate, R_t – nominal interest rate, P_t – deflator (GDP).

Hence, the increase of fixed capital will be equal to the following, where (*) indicates the expected value of variable:

$$\Delta K_t = \gamma K_t^* - \gamma K_{t-1} \tag{3}$$

On the other hand, the increase of fixed capital can be obtained, taking into account the supplies of investment goods understood as a difference between the gross business investment (installed equipment) I_t and liquidation (scrapping) of fixed capital D_t:

$$\Delta K_t = I_t - D_t, \tag{4}$$

Generally it is assumed that the rate of scrapping is constant and it is frequently substituted by the rate of depreciation. We have:

$$D_t = \delta K_{t-1} \tag{5}$$

where δ – depreciation ratio.

Hence $\Delta K_t = I_t - \delta K_{t-1}$ (6)

Comparing equations (4) and (6) we obtain:

$$\gamma K_t^* - \gamma K_{t-1} = I - \delta K_{t-1}.$$

Solving this equation for the gross investment, its demand will be obtained as a function of the demand for fixed capital:

$$I_t = \gamma K_t^* + \left(\delta - \gamma\right) K_{t-1} \tag{7}$$

where: K_t^* is determined from equation (2).

Most important in this chain of relationships is the impact of output, that is the essence of the accelerator rule.

In the macromodels it is generally assumed, that gross investment is equal to the real investment spending J_t $(J_t \equiv I_t)$. However, several country models like W8 models for Poland show differences between the above investment indicators (that happens if the investment expenditures are financing the production in progress). In this case additional bridge equations are introduced:

$$J_t = \sum_{i=0}^{m} \omega_i I_{it}, \tag{8}$$

where: m – number of investment indicators, $0 < \omega_i < 1$ are weights.

The specification of business investment functions in the macroeconometric models often differs from the described above. It will be shown below.

Many model builders linearize the functions determining the demand for fixed capital. It has then the following form (cf. i.a. Klein et al., 1999):

$$K_t^* = \beta_0 + \beta_1 X_t + \beta_2 (R_t / P_t) + \varepsilon_t. \tag{9}$$

The impact of employment was omitted as it was partially captured by the changes in output X_t, and also the effects of technical progress.

Assuming an adaptive adjustments mechanism:

$$K_t = \lambda K_t^* + (1 - \lambda) K_{t-1}$$

we then have

$$\Delta K_t = \lambda \beta_0 + \lambda \beta_1 X_t + \lambda \beta_2 (R_t / P_t) - \lambda K_{t-1} \tag{10}$$

Using next the identity (6) after making use of equations (10) and (6) the equation explaining investment I_t will be derived:

$$I_t = \lambda \beta_0 + \lambda \beta_1 X_t + \lambda \beta_2 (R_t / P_t) + (\delta - \lambda) K_{t-1} \tag{11}$$

The above equation or its logarithmic representation was used in many, mainly early macroeconometric models. It was, in general, extended by introducing relevant lag distributions. The introduction of lag distributions of output i.e. of flexible accelerator was intended to allow for lags in deliveries, installations etc. and also for investors expectations based on the past experience.

In the 60s D. Jorgenson generalized the above model, introducing to the investment function as explanatory variable – broadly understood investment user costs KU_t. He also accentuated that the investment process takes time, that necessitates the introduction of relevant lag distributions (Jorgenson, 1965).

The investment user cost variable was constructed in such a manner, as to include the fiscal components, allowing for analyses of likely impacts of fiscal policy. We have:

$$KU_t = \frac{P_{jt}}{P_t}\left(r_t - \delta\right)\left(1 - ti_t - z_t\right) / \left(1 - t_t\right) \tag{12}$$

where:

P_{jt} are prices of investment goods,

P_t is the GDP deflator,

r_t is the interest rate,

δ is the rate of depreciation,

ti_t is the tax rate of investment credit,

z_t is the tax rate on the depreciation,

t_t is the tax rate on the profits of corporations.

Taking into account the lags in the investment process, we obtain:

$$I_t = \sum_{J=0}^{J} \alpha\beta_j \Delta\left(X_{t-j} KU_{t-j}^{-\sigma}\right) + u_t \tag{13}$$

where σ is the elasticity of substitution between the fixed capital and the remaining production factors and u_t is disturbance term. In applications it is frequently assumed that $\sigma = 1$.

Sometimes, the ratios of investment and fixed capital $\left(i_t - k_{t-1}\right)$ are explained. They depend on the above listed factors (cf. Dreger, Marcellino, 2007).

The neoclassical concept of D. Jorgenson was applied in many macroeconometric models. However, in some of them its simplified versions were used. They included – a) treating the explanatory variables as separable, then b) substituting the sums of first differences ΔX_t by weighted sum of lagged investment I_{t-i} (using the Koyck's transformation) and increase of current output ΔX_t and c) also allowing for the irregularity of the output increases – substituting them by the output levels X_t. The user costs were frequently represented by their major components – interest rates. Taking into account the above mentioned simplifications the long term investment function becomes:

$$I_t^* = \alpha_0 + \alpha_1 I_{t-1} + \alpha_2 X_t + \alpha_3 KU_t + \varepsilon_t. \tag{14}$$

Frequently, in many macroeconometric models the exponential representations are prevailing. Then we have (small letters stay for logarithms):

$$i_t^* = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 x_t + \alpha_3 ku_t + \zeta_t. \tag{15}$$

In several macroeconometric models attempts were made to extend the above specifications by explicit introduction of the rates of capacity utilization WX_t; their high levels induce the enterprises to start new investment projects. These attempts come back to the early, quarterly Wharton models, where the special Wharton capacity utilization indices were used (Evans, Klein, 196; cf. also Harrison et al., 2005; Welfe, 2009). In this case we have:

$$i^* = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 x_t + \alpha_3 ku_t + \alpha_4 wx_t + \zeta_t. \tag{16}$$

At the end of the 60s it was pointed out, that investment process implies the additional installation costs (equipment changes, personal education etc.). Hence, it was suggested to extend the specification of the investment function by adding variable KA_t representing the installation costs.

Following the proposals of Lucas (1967) and Treadway (1969) it was assumed, that the installation costs are a quadratic function of the difference between the investment-fixed capital ratio and its long-run level:

$$KA_t = \frac{\chi}{2}\left[\frac{I_t}{K_{t-1}} - (\delta + g)\right]^2 K_{t-1} \tag{17}$$

where:

δ is the rate of depreciation or scrapings.

g is the long-run rate of growth of the GDP.

In a simplified version the installation costs depend only on the investment-fixed capital ratio, like for instance in the QUEST II model (Roger, int'Veld, 1997).

In several macromodels, especially if they distinguish the investment demand for machinery and equipment like the W8 models for Poland, the substitution of labour for fixed capital is taken into account. To allow for the impact of substitution additional explanatory variable is introduced, representing the ratio of average wages $\left(WP_t\right)$ to investment expenditures deflator $\left(PJ_t\right)$. We have then the following long-term equation, ignoring the installation costs:

$$i_t^* = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 x_t + \alpha_3 ku_t + \alpha_4 wx_t + \alpha_5 \left(wp_t - p_t j\right) + \xi_t \tag{18}$$

and the short-run equation:

$$\Delta i_t = \beta_1 \left(i_{t-1} - i_{t-1}^*\right) + \beta_2 \Delta x_t + \beta_3 \Delta ku_t + \beta_4 \Delta wx_t + \beta_5 \Delta\left(w_t - p_t j\right). \tag{19}$$

The parameters of the above investment demand function for machinery and equipment were estimated for Poland, using the W8 D-2007 model (Welfe, 2009). The following estimates were obtained for the long-run:

$$\hat{\alpha}_2 = 1, \quad \hat{\alpha}_3 = -0.1, \quad \hat{\alpha}_4 = 1.1, \quad \hat{\alpha}_5 = 0.5$$

whereas for the short-run:

$$\hat{\beta}_1 = -0.42 \quad \hat{\beta}_2 = 2.0 \quad \hat{\beta}_3 = 0.56 \quad \hat{\beta}_5 = 0.14,$$

all being statistically significant, including the positive impact of substitution.

The above specifications hardly detected the large fluctuations of investments largely exceeding the variation in the GDP rates of growth. These differences could be attributed to the changing uncertainties in the investment decision process. The investors' expectations were related to the expected investment risks in the capital markets related to globalization.

Hence, in many models a new variable was introduced representing the risk premium, that was defined in many different ways. Introducing this variable we have in the long-run:

$$i^* = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 x_t + \alpha_3 ku_t + \alpha_4 wx_t + \alpha_5 \left(wp_t - p_t j\right) + \alpha_6 rp_t + \zeta_t, \tag{20}$$

where rp_t – risk premium

In several cases, this variable is treated as a component of the user costs.

The second major tendency in modelling investment demand was based on the Q theory developed by J. Tobin (1969). Following this theory, it is assumed that the investment decisions are taken by the enterprises only if the market value of the firm exceeds the replacement costs of its fixed capital.

The above concept, despite its theoretical attractiveness (i.a. it defines the markets expectations) found a restricted use. Especially for Poland it was found that the number of firms registered at the Warsaw Stock Exchange Board is not large enough to undertake this type of study.

In the early stages of modelling the investment demand the importance was attached to the role in the investment decision making process of the availability of investment financing sources. It led to the introduction of financial constraints into the investment demand equations. The investment financing sources covered the firms' own (depreciation and partly profits) and borrowed sources (bank credits) and subsidies. In the models for developed market economies these financial constraints were abandoned, as it was assumed that the effective investment demand has the chances to be fully met and the financial constraints are well represented by the interest rates charging credits. In the emerging markets and developing countries an important role is played by the foreign direct investment (FDI). A certain portion of it is devoted to financing the new "green field" investment products. Hence, the FDI is frequently introduced as an additional variable.

Some revival of these concepts have taken place in recent years, when following the neokeynesian theory it was stressed, that there exists an asymmetry of information available to the banks and firms. In the investment decision making the banks are usually less informed, which makes them suspicious and may lead to the credit reglamentation. They tend to be better secured because of likely losses. That strengthens the need to introduce the investment risk premium as additional variable in the investment demand equations.

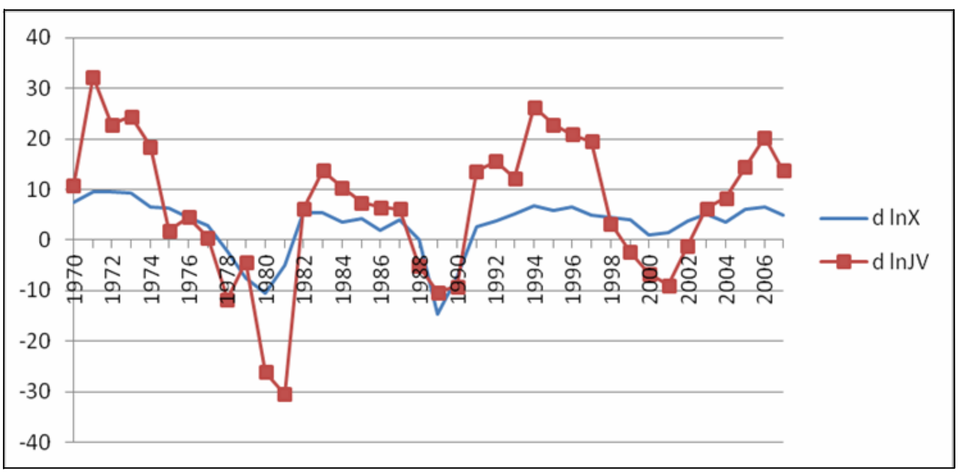
2. EMPIRICAL RESULTS

The investment equations for Poland are based on specification (15). Hence, in order to explain the demand for investment we considered the following explanatory variables: GDP (X_t), the proxy for the rate of capacity utilization (WN_t), the ratio of average wages to the appropriate investment expenditures deflator ($WBP_t/8291/PJV_t$ in equation explaining demand for investment in machinery and equipment JV_t and $WBP_t/8291/PJJT_t$ in equation explaining demand for buildings and structures $JJTF_t$), the real investment user costs ($((1+RKFR_t)/(PJA_t/PJA_{t-1}))*(PJV_t/PX_t)$ in equation for JV_t and

$((1+RKFR_t)/(PJA_t/PJA_{t-1}))*(PJJT_t/PX_t)$ in equation for $JJTF_t$). The real refinancing rate $(1+RKFR_t)/(PJA_t/PJA_{t-1})$ is the main component of the real investment user costs. In the equation explaining demand for investment in machinery and equipment we additionally introduced the variable, which reflects the impact of foreign direct investments ($SJBUSD_t$). Due to large fluctuations of investment expenditure in machinery and equipment and in buildings and structures (see graphs 1 and 2) the new variable that represents the risk premium was introduced. We considered the following risk indicators (in percentage): the ratio of total domestic debt to GDP, the ratio of short-term domestic debt to GDP, the ratio of long-term domestic debt to GDP, the ratio of total foreign debt to GDP, the ratio of long-term foreign debt to GDP, the ratio of the total debt (the sum of domestic and foreign debts) to GDP and the ratio of the Polish budget balance to GDP. In both equations the best results were obtained by introducing the ratio of the Polish budget balance to GDP from previous period ($BDPR_{t-1}$). We expected the positive sign of the parameter associated with that variable because when $BDPR_{t-1}$ rises, the Polish budget deficit declines, which leads to the decrease of investment risk.

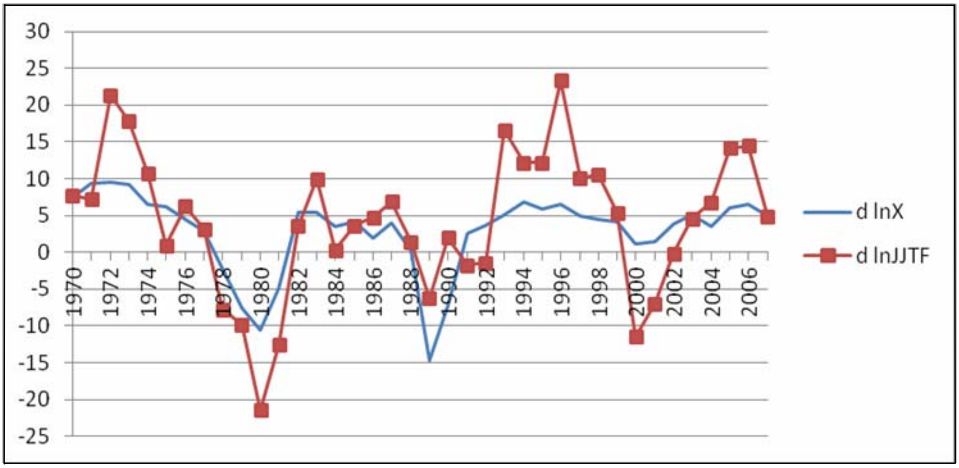
Both explained variables (JV_t and $JJTF_t$) are integrated in order one. The results of ADF test for all variables are presented in table 1. Due to nonstationarity of most variables, the error correction model and the two-stage Engle-Granger estimation method were used.

Graph 1. The rates of growth for GDP and investments in machinery and equipment



Source: own computations, data obtained from GUS (Central Statistical Office).

Graph 2. The rates of growth for GDP and investments in buildings and structures



Source: own computations, data obtained from GUS.

Table 1. ADF test

Variable		ADF		Conclusion
		Test statistic	Critical value	
$\ln X_t$	Level	–1.769	–3.540	I(1)
	Δ	–3.233	–1.950	
$\ln JJTF_t$	level	–2.618	–3.537	I(1)
	Δ	–2.602	–1.950	
$\ln JV_t$	level	–3.043	–3.537	I(1)
	Δ	–2.278	–1.950	
$\ln(WBP_t / 8291 / PJJT_t)$	level	–2.174	–3.533	I(1)
	Δ	–7.430	–2.943	
$\ln(WBP_t / 8291 / PJV_t)$	level	–2.598	–3.674	I(1)
	Δ	–6.181	–3.030	
$\ln(((1 + RKFR_t)/(PJA_t / PJA_{t-1}))*$ $*(PJV_t / PX_t))$	level	–3.920	–3.674	I(0)
	Δ	–	–	
$\ln(((1 + RKFR_t)/(PJA_t / PJA_{t-1}))*$ $*(PJJT_t / PX_t))$	level	–4.271	–3.030	I(0)
	Δ	–	–	
$\ln WN_t$	level	–4.252	–3.674	I(0)
	Δ	–	–	
$\ln(SJBUSD_t)$	level	–8.658	–3.030	I(0)
	Δ	–	–	
$BDPR_t$	level	–2.998	–3.533	I(1)
	Δ	–6.223	–2.943	

Source: own computations.

The long-term equation for JV_t is as follows:

$$\begin{aligned}\ln JV_t = {} & \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln WN_t + \alpha_3 \ln(WBP_t / 8291 / PJV_t) + \\ & + \alpha_4 \ln(((1 + RKFR_t)/(PJA_t / PJA_{t-1})) * (PJV_t / PX_t)) * (1 - U7089_t) + \\ & + \alpha_5 \ln(SJBUSD_t) * (1 - U7089_t) + \alpha_6 BDPR_{t-1} + \\ & + \alpha_7 U71 + \alpha_8 U7476 + \alpha_9 91 + \alpha_{10} 9699 + \varepsilon_t\end{aligned} \tag{21}$$

The estimation results for the equation (21) are presented in table 2.

Table 2. Parameter estimates of long-term equation for investment in machinery and equipment

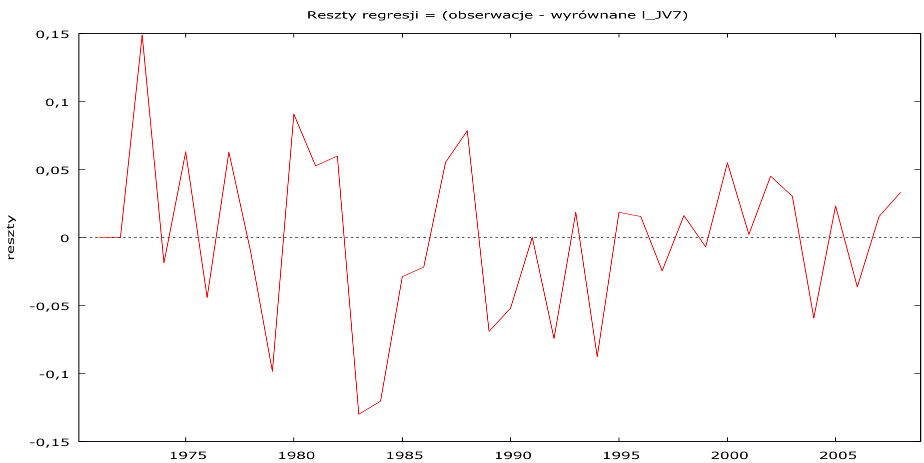
Parameter	estimate	t-statistic	p-value
$\hat{\alpha}_0$	–3.20229	–75.21	8.47e–035 ***
$\hat{\alpha}_1$	1.00000	NA	NA
$\hat{\alpha}_2$	1.00000	NA	NA
$\hat{\alpha}_3$	0.215522	4.673	6.30e–05 ***
$\hat{\alpha}_4$	–0.144396	–2.232	0.0335 **
$\hat{\alpha}_5$	0.0635145	7.831	1.23e–08 ***
$\hat{\alpha}_6$	0.0672077	10.11	5.11e–011 ***
$\hat{\alpha}_7$	–0.215849	–2.8488	0.00799 ***
$\hat{\alpha}_8$	0.311023	7.1517	<0.00001 ***
$\hat{\alpha}_9$	–0.303503	–3.9512	0.00046 ***
$\hat{\alpha}_{10}$	0.188464	4.3366	0.00016 ***
H_0: $\alpha_1 = 1$. $\alpha_2 = 1$. H_1:~ H_0 F(2, 27) = 1.21215, p–value = 0.31324			
H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = \alpha_9 = \alpha_{10} = 0$. H_1:~ H_0 F(8, 29) = 150.0913. p–value = 1.76e–21			
$R^2 = 0.976418$	$\bar{R}^2 = 0.969912$	Durbin–Watson = 2.066611	

Source: own computations.

In long-term equation for JV_t we assume that the elasticities associated with GDP and the rate of capacity utilization are equal to 1. According to test F we cannot reject H_0, which means, that the above restrictions are true. It is worth stressing that there is substitution between labor and fixed capital ($\hat{\alpha}_3 = 0.22$). Estimates of parameters associated with investment user cost, direct foreign investments and risk premium are compliant with our expectations. On the basis of the tests presented in table 2 impacts of all variables are significant and statistical properties of residuals are satisfactory (no autocorrelation) with high determination coefficient.

Residuals from long-term equation for JV_t are presented in graph 3. In order to test stationariry of residuals from long-term equation the cointegrating test CRDF was used. The hypothesis H_0 can be rejected, so the residuals are stationary and the error correction model (22) can be used.

Graph 3. Residuals from long-term equation for JV_t



Source: own computations, using GRETL.

Short-term equation JV_t is as follows:

$$\begin{aligned}\Delta \ln JV_t = {} & \alpha^*(\ln JV_{t-1} - \ln JV^*_{t-1}) + \beta_1 \Delta \ln X_t + \beta_2 \Delta \ln WN_t + \\ & + \beta_3 \Delta \ln(WBP_t / 8291 / PJV_t) + \\ & + \beta_4 (\ln(((1 + RKFR_t)/(PJA_t / PJA_{t-1})) * (PJV_t / PX_t)) * (1 - U7089_t) + \\ & \text{-} \ln(((1 + RKFR_{t-1})/(PJA_{t-1} / PJA_{t-2})) * (PJV_{t-1} / PX_{t-1})) * (1 - U7089_{t-1})) \\ & + \beta_5 (\ln(SJBUSD_t) * (1 - U7089_t) - \ln(SJBUSD_{t-1}) * (1 - U7089_{t-1})) + \\ & + \beta_6 \Delta BDPR_{t-1} + \beta_7 U7274_t + \beta_8 U7677_t + \beta_9 U8283_t + \\ & + \beta_{10} U96_t + \beta_{11} U00_t + \varepsilon_t ,\end{aligned} \tag{22}$$

where $\ln JV^*_{t-1}$ stands for theoretical values from long-term equation.

We observe moderate speed of adjustments to the long run equilibrium (the error correction term is equal to –0,54). Short-term elasticity associated with GDP is almost 2, which means that increment of investment is faster that increment of GDP. In the short-term equation we also assume rate of capacity utilization equal to 1 and according to test F we can accept the imposed restriction. The impact of the increment of ratio of average wages to the investment deflator turned out to be insignificant. The other estimates are compliant with expectations (see table 3).

Table 3. Parameter estimates for short-term equation for investment in machinery and equipment

parameter	estimate	t-statistic	p-value
$\hat{\alpha}$ *	–0.539110	–3,475	0,0017 ***
$\hat{\beta}_1$	1.91509	9,796	2,21e-010 ***
$\hat{\beta}_2$	1.00000	NA	NA
$\hat{\beta}_4$	–0.0982624	–2,431	0,0220 **
$\hat{\beta}_5$	0.0481827	3,967	0,0005 ***
$\hat{\beta}_6$	0.0148374	2,157	0,0401 **
$\hat{\beta}_7$	0.117661	3,2176	0,00335 ***
$\hat{\beta}_8$	–0.0928358	–2,4449	0,02130 **
$\hat{\beta}_9$	–0.0976901	–2,6441	0,01348 **
$\hat{\beta}_{10}$	0.114878	2,0962	0,04557 **
$\hat{\beta}_{11}$	–0.129509	–2,4982	0,01887 **
H_0: $\beta_2 = 1$, H_1:~ H_0 F(1, 26) = 0.300107, p-value = 0.588488			
H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = \beta_{11} = 0$, H_1:~ H_0 F(10, 27)= 29.84055, p-value = 4.99e-12			
$R^2 =$ 0.917027	$\bar{R}^2 =$ 0.889369	Durbin-Watson = 1.994412	

Source: own computations.

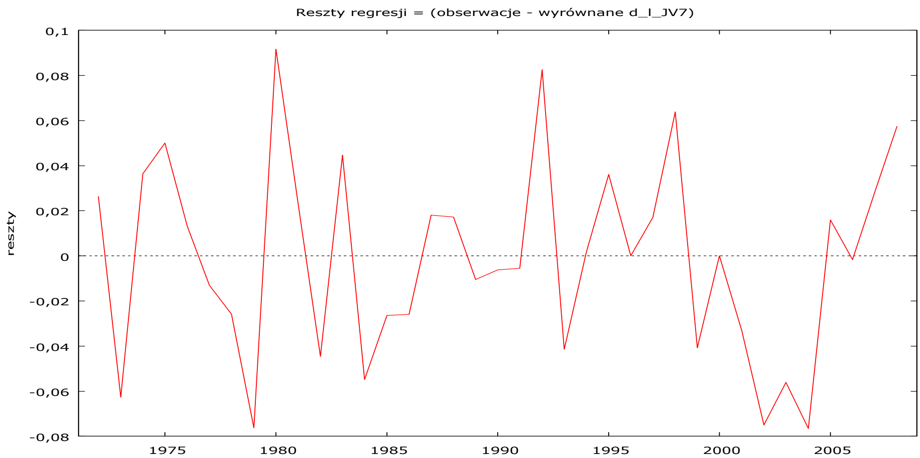
Residual analysis includes tests on heteroscedasticity, normality, autocorrelation and stability of parameters (see table 4). We can conclude, that in the long-term as well as in the short-term equation of the demand for investment in machinery and equipment the residuals are normally distributed, homoscedastic, without autocorrelation and parameters from both equations are stable (see graphs 5 and 6).

Table 4. Residual analysis for JV_t

Residual analysis	Long-term equation	Short-term equation
White Test on heteroscedasticity (LM statistic)	0.700694 (p = 0.704444)	0.197088 (p = 0.906156)
Test on normality (Chi-squared statistic)	0.0477093 (p = 0.828683)	0.0209035 (p = 0.886157)
LM test on autocorrelation	11.7989 (p = 0.461962)	21.5206 (p = 0.0890159)
CUSUM test on parameter stability	1.41334 (p = 0.168576)	0.0273865 (p = 0.978361)

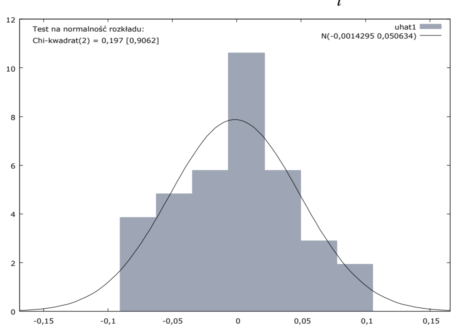
Source: own computations.

Graph 4. Residuals from short-term equation for JV_t



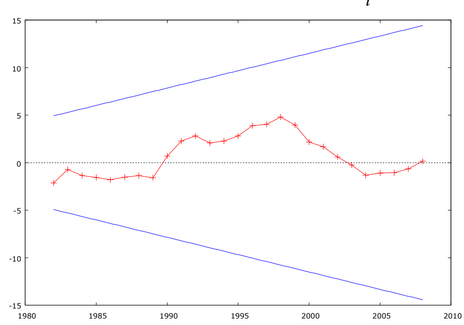
Source: own computations, using GRETL.

Graph 5. Distribution of residuals from error correction model for JV_t



Source: own computations, using GRETL

Graph 6. Test on stability of parameters from error correction model for JV_t



Source: own computations, using GRETL

The long-term equation of the demand for investment in buildings and structures has the following form:

$$\ln JJTF_t = \alpha_0 + \alpha_1 \ln X_t + \alpha_2 \ln(WBP_t / 8291 / PJJT_t) + \\ + \alpha_3 \ln(((1 + RKFR_t)/(PJA_t / PJA_{t-1})) * (PJJT_t / PX_t)) * (1 - U7089_t) + \\ + \alpha_4 BDPR_{t-1} + \alpha_5 U7376_t + \alpha_6 U81_t + \alpha_7 U9192_t + \alpha_8 U9700_t + \varepsilon_t \quad (23)$$

Table 5. Parameter estimates for long-term equation for investment in buildings and structures

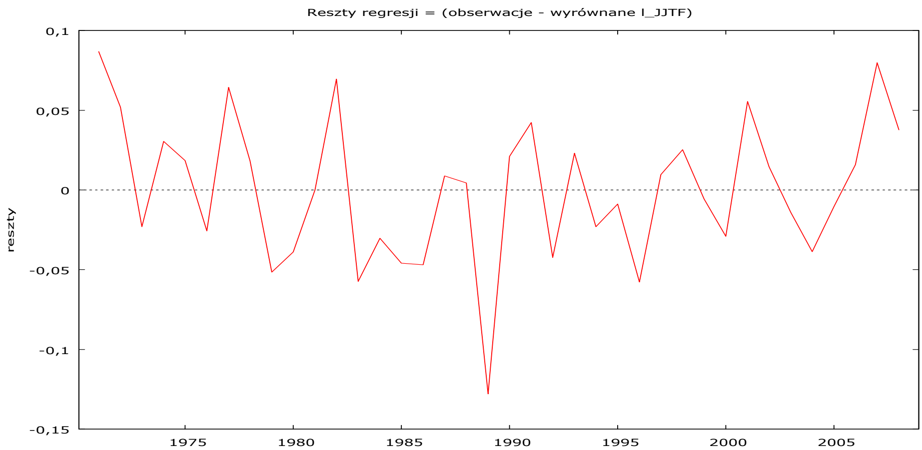
parameter	estimate	t - statistic	p-value
$\hat{\alpha}_0$	–0.286397	–0.2335	0.81700
$\hat{\alpha}_1$	0.858509	9.0576	<0.00001 ***
$\hat{\alpha}_2$	0.421592	9.5137	<0.00001 ***
$\hat{\alpha}_3$	–0.168871	–2.6209	0.01382 **
$\hat{\alpha}_4$	0.031108	6.1020	<0.00001 ***
$\hat{\alpha}_5$	0.177109	6.1855	<0.00001 ***
$\hat{\alpha}_6$	–0.176983	–3.1991	0.00333 ***
$\hat{\alpha}_7$	0.101121	2.1944	0.03637 **
$\hat{\alpha}_8$	0.230962	7.9226	<0.00001 ***
H_0: $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_6 = \alpha_7 = \alpha_8 = 0$, H_1:~ H_0 $F(8, 29) = 324.0097$, p-value = 3.13e-26			
$R^2 = 0.988936$	$\bar{R}^2 = 0.985884$	Durbin-Watson = 1.799936	

Source: own computations.

In the long-term equation for $JJTF_t$ it was not necessary to impose any restrictions (see table 6). Long-term elasticity associated with GDP is less than 1. Furthermore, there exists substantial substitution between labor and capital assets. Estimates of parameters associated with investment user cost and risk premium are compliant with our expectations. Analyzing the results of the tests presented in table 6 we can conclude, that all variables are significant, residuals are not correlated and determination coefficient is almost one.

Residuals from long-term equation for $JJTF_t$ are presented in graph 7. In order to test stationarity of residuals from the long-term equation the cointegrating test CRDF was used. The hypothesis H_0 can be rejected, so the residuals are stationary and the error correction model (24) can be used.

Graph 7. Residuals from long-term equation for $JJTF_t$



Source: own computations, using GRETL.

The short-term (ECM) equation for investment in buildings and structures is as follows:

$$\Delta \ln JJTF_t = \alpha^*(\ln JJTF_{t-1} - \ln JJTF^*_{t-1}) + \beta_1 \Delta \ln X_t + \beta_2 \Delta \ln(WBP_t / 8291 / PJJT_t) +$$
$$+ \beta_3(\ln(((1+RKFR_t)/(PJA_t/PJA_{t-1}))*(PJJT_t/PX_t))*(1-U7089_t)+$$
$$-\ln(((1+RKFR_{t-1})/(PJA_{t-1}/PJA_{t-2}))*(PJJT_{t-1}/PX_{t-1}))*(1-U7089_{t-1}))+$$
$$+\beta_4 \Delta BDPR_{t-1} + \beta_5 U73_t + \beta_6 U7677_t + \beta_7 U91_t + \beta_8 U97_t + \beta_9 U01_t + \varepsilon_t \quad (24)$$

where $\ln JJTF^*_{t-1}$ stands for theoretical values from the long-term equation.

Table 6. Parameter estimates for short-term equation explaining the demand for investments in buildings and structures

Parameter	estimate	t – statistic	p–value
1	2	3	4
$\hat{\alpha}^*$	–0.432071	–2.4679	0.01997 **
$\hat{\beta}_1$	1.40444	10.2694	<0.00001 ***
$\hat{\beta}_3$	–0.171531	–3.1439	0.00392 ***
$\hat{\beta}_4$	0.0199828	4.0186	0.00040 ***
$\hat{\beta}_5$	0.112678	2.8148	0.00883 ***
$\hat{\beta}_6$	–0.0791246	–2.8137	0.00886 ***

Table 6 (cont.)

1	2	3	4
$\hat{\beta}_7$	0.278564	3.7306	0.00086 ***
$\hat{\beta}_8$	0.133707	3.2423	0.00306 ***
$\hat{\beta}_9$	–0.146492	–3.8819	0.00058 ***
H$_0$: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = 0$, H$_1$:~ H$_0$ F(10, 27)= 29.26374, p–value = 6.09e–12			
$R^2 =$ 0.903903	$\bar{R}^2 = 0.876447$	Durbin–Watson = 1.719333	

Source: own computations.

In the error correction model for $JJTF_t$ we also notice moderate speed of adjustments to the long run equilibrium. The short-term elasticity associated with GDP is 1.4, which means that increment of investment is faster that increment of GDP. The impact of the increment of the ratio of average wages to the investment deflator turned out to be insignificant. The other estimates are values compliant with expectations (see table 7).

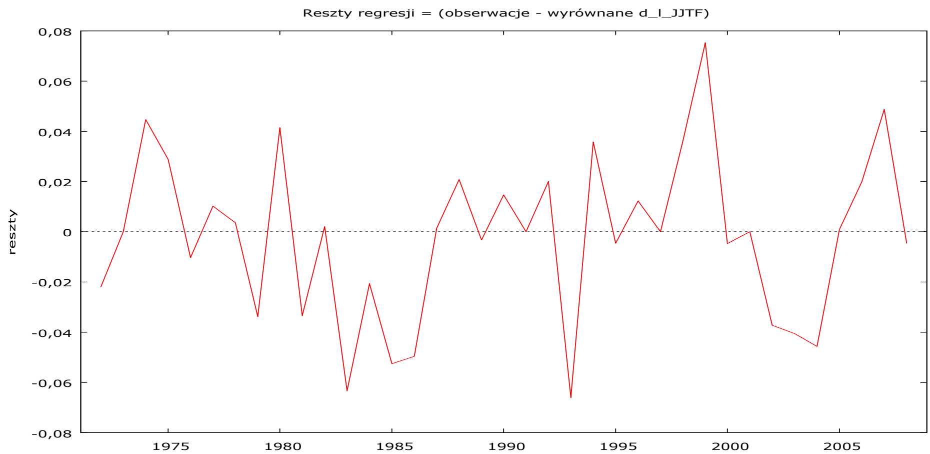
The results of the residual analysis are the same as in the previous model. In the long-term as well as in the short-term equation of the demand for investment in buildings and structures the residuals are normally distributed, homoscedastic, without autocorrelation and parameters are stable (see table 7, graphs 9 and 10).

Table 7. Residual analysis for $JJTF_t$

Residual analysis	Long-term equation	Short-term (ECM) equation
White Test on heteroscedasticity (LM statistic)	9.42449 (p = 0.666309)	15.9175 (p = 0.195042)
Test on normality (Chi-squared statistic)	1.74945 (0.416978)	0.0807971 (p=0.960407)
LM test on autocorrelation	0.0664331 (p = 0.798488)	0.597206 (p=0.446359)
CUSUM test on parameter stability	1.88047 (p = 0.0704843)	0.0614132 (p=0.951483)

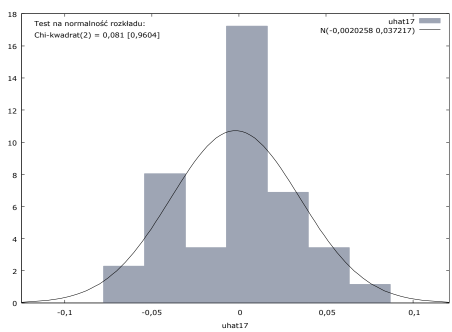
Source: own computations.

Graph 8. Residuals from short-term equation for $JJTF_t$



Source: own computations, using GRETL.

Graph 9. Distribution of residuals from error correction model for $JJTF_t$



Source: own computations, using GRETL

Graph 10. Test on stability of parameters from error correction model for $JJTF_t$

Source: own computations, using GRETL

3. CONCLUSIONS

Introducing new variable – risk premium, represented by the ratio of Polish budget balance to GDP, in investment functions improves empirical results in each case. Previous version of the model contained more restrictions imposed. In the long-term equation for JV_t and in the long-term and in the short-term (ECM) equations for $JJTF_t$ it was necessary to calibrate parameters associated with investment user cost. In the new specification we obtained estimates that are significant and compliant with a priori expectations. Comparison of both specifications is presented in table 8.

In the long term equation explaining investment in machinery and equipment we still had to impose restrictions in both specifications on parameters showing the impact of GDP and its utilization. However, in the new model taking into account the investment risk the marginal level of degrees of freedom of F test occurred much higher than in the old specification. The long-term impact of foreign direct investment is much stronger in the new model. In the short-term equation for JV_t the estimates of parameters are stable, showing that the introduction of the new variable did not considerably change the obtained estimates.

The long-term equation explaining $JJTF_t$ delivered in the old model much higher value of elasticity with respect to GDP than in the new model. The short-term impact turned out also lower in the new model than in the old specification, whereas the effect of labor substitution occurred much stronger in the new model. The error correction terms are in both versions similar, whereas they are much higher for the new model as regards the equation explaining JV_t.

In the new model we observe higher adjusted R squared, higher p-value in F test, which was used in testing the restrictions imposed in equation for JV_t . Furthermore there are better statistical properties for the models presented in this article.

The introduction of the new risk variable resulted in an increase of accuracy – the R squared in each case was higher in the new specification. Other tests confirmed the superiority of the new specification.

Table 8. Comparison of old and new specifications

Machinery and equipment				
Parameter associated with:	Long-run		Short-run	
	W8	New model	W8	New model
1	2	3	4	5
X_t	1 (restricted)	1 (restricted)	2.01	1.92
WN_t	1 (restricted)	1 (restricted)	–	1 (restricted)
WBP_t...	0.4	0.22	0.17	–
$(1+RKFR_t)$...	–0.1 (restricted)	–0.14	–0.08	–0.1
$SJBUSD_t$	0.02	0.06	0.05	0.05
$BDPR_{t-1}$	–	0.07	–	0.01
error correction term	–	–	–0.26	–0.54
Adjusted R^2	0.95	0.97	0.84	0.89

Table 8 (cont.)

1	2	3	4	5
Buildings and structures				
Parameter associated with:	Long-run		Short-run	
	W8	New model	W8	New model
X_t	1.31	0.86	1.78	1.40
WN_t	0.93	–	–	–
WBP_t...	0.09	0.43	–	–
$(1+RKFR_t)$...	–0.1 (restricted)	–0.17	–0.1 (restricted)	–0.17
$BDPR_{t-1}$	–	0.31	–	0.02
error correction term	–	–	–0.53	–0.43
Adjusted R^2	0.99	0.99	0.82	0.88

Source: own computations.

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ABSTRACTS

Modelling Economies in Transition 2011
Łódź 2012, pp. 111–112

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The Investors' Risk Aversion and the Long-Term Economic Growth in a Schumpeterian Framework

Abstract. The class of the Schumpeterian models of economic growth is becoming increasingly popular. After the introduction of some crucial extensions of the basic models by Aghion and Howitt (1992), (1998) many dimensions of the process of economic growth can now be described. The field in which we would like to conduct our reasoning is: defining and describing the relations between capital market and the process of long-term economic growth. We refer to the model by Aghion, Howitt and Mayer-Foulkes (2004) in which financial development in a particular economy (which depicts the level of creditor protection) becomes a necessary condition for the convergence of the country's rate of growth.

In the paper we modified the initial model by Aghion, Howitt and Mayer-Foulkes (2004). We implemented some alternative dynamics into the basic framework. In the first part of our analysis we quantified the probability of default of an enterprise which is trying to introduce an innovation. We showed, how the level of risk in the economy (measured by the enterprises' probability of default) influences the process of gaining the financial capital by entrepreneurs and in what way it stimulates the process of economic growth. In another part of the paper we presented a modification in which we introduced heterogeneous agents on the capital market. The buyers of corporate bonds are now characterized by different parameters describing their risk aversion. This fact causes that not all of the issues of corporate bonds will be placed, which means that some innovation projects will be cancelled. In this model the role of financial authority is crucial, because it shapes the markets' expectations about the future soundness of entrepreneurs.

Considering the two extensions of the model by Aghion, Howitt and Mayer-Foulkes (2004), we answered three question. How the liquidity of capital market contributes to the process of economic growth? Does the distribution of investors' risk aversion influence the pace of economic growth? What is the role of financial authorities in providing high performance of the real economy?

Modelling Economies in Transition 2011
Łódź 2012, p. 113

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External Effects of Industrial Clustering in Poland

Abstract. Clusters and cluster-based policy gained a lot of significance and relevance among policy-makers in recent years. Despite the role of clusters as potential drivers of regional development has not been extensively tested and thus we are left with scant evidence. The present study adopts the general approach of Rodriguez-Pose and Comptour (2010), modifies it and looks at the relationship between the scope and scale of industrial clustering and the growth of Polish NUTS3 regions in the period 2000–2008. We estimate an extended cross-sectional empirical model of growth accounting for major drivers of regional economic development grouped into individual filters as well for overall clusterization index in order to identify the impact of clusters. In the estimation we account for potential spatial dependence of bordering regions as well as agglomeration process along the lines of spatial econometrics methodology. The initial results are promising.

Key words: industrial clustering, economic growth, spatial econometrics, Poland, regional development

Modelling Economies in Transition 2011
Łódź 2012, p. 114

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Consumption Function For Poland. Is Life-Cycle Hypothesis Legitimate?

Abstract. In developed countries the rising percentage of households behave rationally. It is represented by the life-cycle hypothesis of determining their incomes. The W models of the Polish economy assumed that the number of those households is small and this approach was neglected.

The paper presents the results of a research project aimed at empirical testing, whether the share of “rational” households in Poland was small and their majority was income-constrained. We calculated the expected life-cycle income using the information on the structure of employment by age and the sample of 1970–2008. The obtained results show that the share of “rational” households was below 10%.

ECONOMIC GROWTH AND BUSINESS CYCLE

Modelling Economies in Transition 2011
Łódź 2012

Modelling Economies in Transition 2011
Łódź 2012, pp. 117–138

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Investment In Human Capital as the Best Source of Economic Growth after the Adoption of the Euro

Abstract. The paper presents the exogenous growth model of Mankiw, Romer and Weil modified and expanded so as to describe a small economy in an economic and monetary union (EMU). We assume perfect mobility of capital, and immobility of the so-called raw labor (empirically justified in Europe). On the other hand, we assume that human capital (highly skilled and educated individuals) may be mobile. We prove that the unique dynamic equilibrium (the steady state) exists, and is at least locally asymptotically stable. Finally, we derive the golden rules of accumulation of physical and human capital. We prove that the optimal rates of investment in physical and human capital depend upon the natural rate of growth and the real interest rate. If they are equal, there are infinitely many optimal pairs of investment rates. On the other hand, if they differ, the golden rule recommends one of two extreme (edge) solutions. In all three cases, however, the optimal investment rates are linked together by a very simple linear equation (the line H). The economy should always stay on the line H, and move along this line, either up or down, in response to changes in exogenous parameters.

In the second part of the paper, we carry out numerical experiments, based on realistic (calibrated) values of exogenous parameters. Simulations suggest very strongly that current levels of investment in human capital (education) in Poland are way too low. Clearly, there is no better way of promoting economic growth than investing heavily in human capital.

These conclusions are not immune to the increasing mobility of human capital in an EMU. At the final section, we present an extension of the model which allows for different levels of human capital mobility. We demonstrate that low level of mobility of human capital only slightly weakens our conclusions – investing in human capital still remains the best way to improve economic situation in the long-run. However, if the outflow of human capital reaches certain critical level, the economy may fall into the poverty trap.

Key words: human capital, monetary union, golden rule, economic growth
JEL codes: F43, H52, J24, O41

1. CLOSED ECONOMY MANKIW-ROMER-WEIL MODEL (SHORT REVIEW)

The starting point in our analysis is the well-known exogenous growth model by Mankiw, Romer, and Weil (1992). It is a closed economy model, so we will need to modify it so as to describe a small open economy in an economic and monetary union. Naturally, we mean Poland, however the model is general, i.e. it can easily be applied to other countries, as long as their economies are small compared to the entire EMU.

We begin with a brief review of the MRW model. The technology is represented by the three-factor Cobb-Douglas production function with constant returns to scale and labor-augmenting technological progress:

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}, \quad \alpha + \beta < 1, \quad \alpha, \beta > 0 \tag{1}$$

where K is the stock of physical capital in the country, H is the stock of human capital, L is raw labor, and A represents the level of technology. Both L and A grow exponentially at constant (exogenous) rates n and σ, respectively. Both kinds of capital are accumulated through investment:

$$\dot{K} = I_K - \delta K \tag{2}$$

$$\dot{H} = I_H - \delta H \tag{3}$$

For simplicity, we assume equal depreciation rates for both kinds of capital. In the closed economy $Y = C + I = C + S$, hence the disposable income (gross national product, GNP) is equal to the domestic output: $Y_d = Y$. Let γ_K and γ_H be the savings rates, i.e. fractions of income invested in (respectively) K and H:

$$I_K = \gamma_K Y, \quad I_H = \gamma_H Y, \quad \gamma_K, \gamma_H \geq 0, \quad \gamma_K + \gamma_H = \gamma < 1 \tag{4}$$

Of course, $I = I_K + I_H$ and $C = (1-\gamma)Y$.

Now, a few words about the input markets. Thanks to constant returns to scale, we can apply standard aggregation procedure, and treat entire economy as a single (representative) firm in the perfectly competitive market. In the profit maximizing equilibrium, all factors are paid their marginal products, i.e.

$$\alpha(k^D)^{\alpha-1}(h^D)^{\beta} = w_K = r + \delta$$
$$\beta(k^D)^{\alpha}(h^D)^{\beta-1} = w_H \quad (5)$$
$$(1-\alpha-\beta)(k^D)^{\alpha}(h^D)^{\beta} A = w$$

where $k = K / AL$, and $h = H / AL$ are quantities per effective unit of labor; the superscript D refers to the total demand for respective inputs, w stands for the wage rate of raw labor, w_H is the wage rate of human capital. Finally, w_K is the real rental rate of physical capital, which in equilibrium is equal to the sum of the real interest rate and the depreciation rate[1]. The conditions (5) imply that

$$k^D = \frac{\alpha}{1-\alpha-\beta} \cdot \frac{w}{r+\delta} \cdot \frac{1}{A} \quad (6)$$

$$h^D = \frac{\beta}{1-\alpha-\beta} \cdot \frac{w}{w_H} \cdot \frac{1}{A}. \quad (7)$$

The above ratios of physical and human capital to effective labor determine the optimal ray in the 3-dimensional space of inputs, as illustrated in fig. 1.

If the wage rates of all three inputs are perfectly elastic (standard assumption in closed economy growth models), then the input markets are in permanent equilibrium, i.e. $K^D = K$, $H^D = H$, and $L^D = L$, where K, H, and L represent a given supply (stocks of inputs at a given moment). Under this assumption, the real wage rates and the real interest rate behave according to:

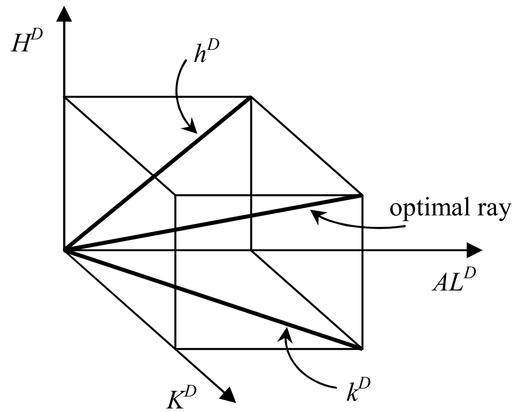
$$r = \alpha k^{\alpha-1} h^{\beta} - \delta \quad (8)$$

$$w_H = \beta k^{\alpha} h^{\beta-1} \quad (9)$$

$$w = A(1-\alpha-\beta) k^{\alpha} h^{\beta} \quad (10)$$

[1] This condition implies that in equilibrium capital owners are indifferent between allocating their assets in risk-free bonds and investing in physical capital, which after depreciation yields the same real rate of return r.

Fig. 1. The optimal ray in the space of production factors



2. A SMALL ECONOMY IN AN EMU

An economic union means free trade, as well as free movement of all factors of production. On the other hand, monetary union means that all member countries use the same currency, and (more importanty) monetary policy is the hands of a single central bank. We also assume that the home country is small in comparison with the entire union. Any changes in our economy have no influence on the union-wide (average) level of prices, wage rates, interest rates etc. We will also abstract from the outside world, so the terms "abroad" or "foreign" refer to "all other EMU countries".

Since goods move freely within EMU, price levels and inflation rates in all member countries are equal. We admit that this is a very strong assumption[2], and in reality price convergence is a very long process. The central bank sets identical nominal interest rates for all EMU countries, and – due to identical inflation rates – real interest rates are also uniform across EMU. From the point of view of any small country, the real interest rate is exogenous: $r = r*$.

Due to perfect capital mobility, the supply of capital in a small economy instantaneously adjusts to demand. Let K be the domestic capital (the stock of productive capital located within the country). A certain part of K is owned by foreigners. Let KN be the national capital, i.e. capital owned by citizens of our country. A part of KN is located within the country, and the rest is invested

[2] For example, EBC (2005) concludes that differences in prices in EU countries have generally fallen and are similar to those observed in the U.S. Some of the latest publications, however, cast doubts. See for example Parsley and Wei (2008), or Wolszczak-Derlacz (2010).

abroad (see table 1). Let E be the net foreign assets, i.e. $E = KN_A - KF_D$. Obviously, $KN = K + E$.

Table 1. The structure of capital in an EMU

owned by: \ located	domestically (K)	abroad
citizens of the country (KN)	KN_D	KN_A
foreigners (KF)	KF_D	KF_A

KN – national capital, KF – foreign capital, KN_D – national capital located domestically, KN_A – national capital located abroad, KF_D – foreign capital located in the country, KF_A – foreign capital located abroad (outside of the country)

Raw labor L and human capital H are immobile – every country can only use for production its own stocks of L and H. This assumption has some empirical support[3]. In one of the final sections of the paper we will allow for some degree of mobility of human capital, and see how it changes our conclusions.

Finally, in accordance with empirical research (Barro et al., 1995), we assume that human capital investment is financed exclusively by domestic savings.

Now, a few words about the national income creation and distribution. We don't distinguish the public sector (government), hence the GDP is defined as:

$$Y = C + I + X - M . \tag{11}$$

The domestic investment comparises two types of capital:

$$I = I_K + I_H . \tag{12}$$

The national income (we also call it the disposable income Y_d) is equal to the GDP plus earnings from net foreign assets (the real rental rate $w_K = r + \delta$, and it is identical across EMU):

$$Y_d = Y + (r + \delta)E . \tag{13}$$

[3] Within the EU mobility of people is low, compared to the U.S. Language and cultural barriers constitute serious obstacles to migration within Europe. See Bertola (1999), Krueger (2000).

The national income is partly consumed; whatever remains is saved: $Y_d = C + S$. The current account balance is defined as $Q = X - M + (r+\delta)E$. Of course $Q = S - I$. Let γ_K and γ_H be the fractions of income saved and invested in (respectively) *KN* and *H*:

$$S = \gamma_H Y_d + \gamma_K Y_d, \quad \text{where} \quad \gamma = \gamma_H + \gamma_K < 1, \ \gamma_H, \gamma_K \geq 0. \tag{14}$$

Obviously, $C = (1-\gamma)Y$. Physical and human capital accumulation is described by:

$$\dot{H} = \gamma_H Y_d - \delta H \tag{15}$$

$$\dot{KN} = \gamma_K Y_d - \delta KN. \tag{16}$$

It follows from the above equations that

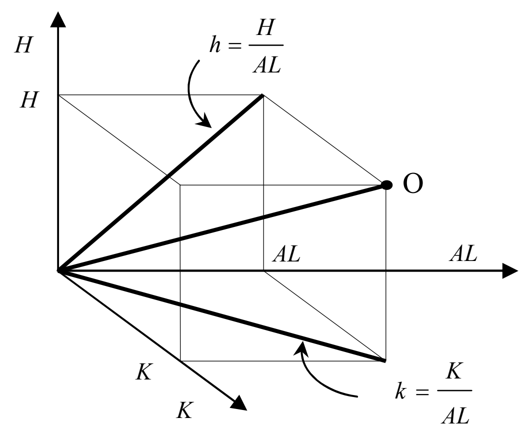
$$\dot{E} = \dot{KN} - \dot{K} = \gamma_K Y_d - \delta KN - (I_K - \delta K) = \gamma_K Y_d - I_K - \delta(KN - K) = Q - \delta E \tag{17}$$

hence the evolution of net foreign assets is determined by the path of current accounts.

In profit-maximizing equilibrium, all three factors of production are paid their marginal products, i.e. equalities (5) hold. The optimal ray determined by (6) and (7) looks just like in the MRW model (fig. 1), but there is one important difference. In the MRW closed economy model all three markets of production factors are in permanent equilibrium, i.e. $K^D = K$, $H^D = H$ and $L^D = L$ (where *K*, *H* and *L* represent stocks at any given moment) thanks to perfect elasticity of wages *w* and w_H coupled with perfectly elastic interest rate *r*. After its entry into EMU, a small economy loses elastic interest rate (it turns exogenous: $r = r^*$), but instead gains perfect capital mobility. Therefore, demand for capital within the country is once again equal to supply ($K^D = K$) – the only difference being perfectly elastic supply (instantaneous outflow/inflow of capital from abroad).

These adjustment mechanisms can be illustrated in the 3-dimensional space of inputs. Let's start with the closed economy MRW model. At any given moment the stocks of *AL*, *H* and *K* are fixed (fig. 2, point O). The demand is described by equations (5) determining the optimal ray. The general equilibrium between supply and demand requires that the optimal ray goes through a given point O. It is so thanks to perfectly elastic prices (*w*, w_H, *r*) which render a position of the optimal ray perfectly elastic. In other words, (by assumption) both wage rates and the real interest rate adjust instantaneously so as to make the optimal ray go through point O. The equilibrium values of wages and the interest rate are given by (8) – (10).

Fig. 2. The optimal ray of production factors in the MRW closed economy model

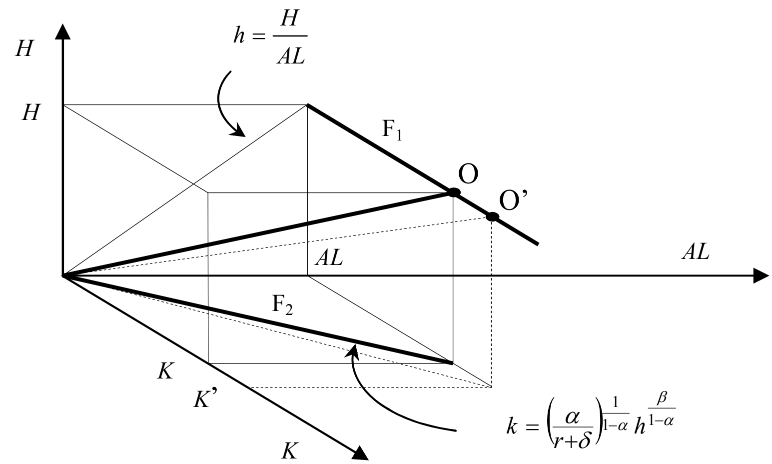


In the case of a small economy in EMU, at any given time only stocks of *AL* and *H* are fixed, hence general equilibrium requires that the optimal ray intersects with the straight line F_1 (fig. 3). On the other hand, it follows from (5) that

$$k = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} h^{\frac{\beta}{1-\alpha}} \tag{18}$$

with $r = r^*$. The equation (18) with its right-hand side fixed at any given moment determines another straight line F_2. Therefore, equilibrium requires that the optimal ray not only intersects with the line F_1, but also lies exactly above the F_2 line. Clearly, there exists only one ray which satisfies both conditions. Any change in the foreign interest rate r^* shifts the F_2 line. For example, a reduction of r^* shifts F_2 outwards (dotted lines in fig. 3). The equilibrium between capital supply and demand is restored through an inflow of capital from abroad.

Fig. 3. The optimal ray of production factors in a small economy in an EMU



3. THE MODEL AND ITS DYNAMICS

The model can be summarized as follows:

[i] $k = \left(\dfrac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} h^{\frac{\beta}{1-\alpha}}$ $\alpha, \beta > 0$ and $\alpha + \beta < 1$

[ii] $y = k^{\alpha} h^{\beta}$

[iii] $y_d = y + (r+\delta)e$

[iv] $c = (1 - \gamma_K - \gamma_H) y_d$ $\gamma_K, \gamma_H \geq 0$, $\gamma_K + \gamma_H \in [0,1)$

[v] $y = c + i + x - m$

[vi] $i = i_K + i_H = i_K + \gamma_H y_d$

[vii] $q = x - m + (r+\delta)e$

[viii] $\dot{k} = i_K - \varphi k$ $\varphi = n + \sigma + \delta$

[ix] $\dot{h} = \gamma_H y_d - \varphi h$

[x] $\dot{e} = q - \varphi e$

with initial (endowment) conditions: $h(0) = h_0 > 0$, $e(0) = e_0$. There are 10 endogenous variables: *k*, *y*, y_d, *c*, $x - m$, *i*, *h*, *q*, i_K, *e*. The differentiation of [i] with respect to time yields:

$$\dot{k} = N \cdot \dot{h} \cdot h^{\frac{\alpha+\beta-1}{1-\alpha}}, \quad \text{where} \quad N = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} \cdot \frac{\beta}{1-\alpha} = const. > 0 \qquad (19)$$

Equations (19), [viii] and [ix] imply that

$$i_K = N \cdot h^{\frac{\alpha+\beta-1}{1-\alpha}} \left[\gamma_H y_d - \varphi h\right] + \varphi k \,. \tag{20}$$

By making use of (19) and (20) we can write the model in the recursive form (which is very convenient for numerical simulations):

[k] $\quad k = \left(\dfrac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}} h^{\frac{\beta}{1-\alpha}}$,

[y] $\quad y = k^{\alpha} h^{\beta}$,

[y_d] $\quad y_d = y + (r+\delta)e$,

[c] $\quad c = (1-\gamma_K-\gamma_H) y_d$,

[i_K] $\quad i_K = N \cdot h^{\frac{\alpha+\beta-1}{1-\alpha}} \left[\gamma_H y_d - \varphi h\right] + \varphi k$,

[$x-m$] $\quad x - m = y - c - \gamma_H y_d - i_K$,

[q] $\quad q = x - m + (r+\delta)e$,

[h] $\quad \dot{h} = \gamma_H y_d - \varphi h$,

[e] $\quad \dot{e} = q - \varphi e$,

with the same initial conditions: $h(0) = h_0 > 0$, $e(0) = e_0$.

4. THE STEADY STATE AND ITS STABILITY

The steady state can be defined by two conditions: $\dot{h} = 0$, and $\dot{e} = 0$. In Konopczyński (2009) we prove that it exists and is unique, and we derive the steady-state consumption:

$$\bar{c} = (1-\gamma_K-\gamma_H) \cdot \frac{(1-\alpha)\varphi}{\varphi - \gamma_K (r+\delta)} \bar{y} \,, \tag{21}$$

where $\bar{y} = M^{\frac{\beta}{1-\alpha-\beta}} \left(\dfrac{\alpha}{r+\delta}\right)^{\frac{\alpha+\beta}{1-\alpha-\beta}}$ and $M = \dfrac{1-\alpha}{\alpha} \cdot \dfrac{\gamma_H (r+\delta)}{\varphi - \gamma_K (r+\delta)}$ (22)

Proposition 1.[4]
The steady state is locally asymptotically stable if and only if

$$r < \frac{\varphi}{\gamma_K} - \delta . \tag{23}$$

This (necessary and sufficient) stability condition is usually satisfied in the real world. To see why, let us carry out a little mathematical experiment. Lets assume some realistic values of the parameters on the right-side of (23). Let $n = 0\%$ (constant population), $\sigma = 3\%$, and $\delta = 4\%$. The realistic value of γ_K could be around 20%. Then the right-hand side of the inequality (23) is equal to 31%. Thus in this realistic example the condition (23) is violated only if the real interest rate exceeds 31%, which is virtually never observed in real economies (perhaps only if the country is on the verge of bankruptcy).

5. THE GOLDEN RULE

The golden rule in growth models refers to the recipe for the highest consumption in the long-run equilibrium (the steady state). In the classic MRW model of the closed economy, the golden rule is very simple. It states that optimal investment rates in physical and human capital should be equal to the elasticities of the production function with respect to physical capital and human capital (α and β), respectively. Empirical data suggests that in most countries, on average $\alpha = 1/3$, and $\beta = 1/3$. Hence, in order to maximize consumption 1/3 of national income should be invested in physical capital, another 1/3 should be devoted to human capital, and the rest can be consumed.

This rule was developed for closed economy. In a small economy under monetary union this rule becomes entirely different. It can be proved that there exist infinitely many pairs of optimal investment rates, but they are always bound together by a linear equation. The golden rule recommends two entirely different strategies of investing into physical and human capital, depending on the relation between the natural rate of growth and the real interest rate.

Proposition 2.
The golden rule has the following mathematical form:

(a) If $r > n + \sigma$, then $\gamma_K^{opt} \to \frac{n+\sigma+\delta}{r+\delta}$ and $\gamma_H^{opt} \to \frac{\beta}{(1-\alpha)} \frac{r-n-\sigma}{r+\delta}$.

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[4] Proved in Konopczyński (2009).

(b) If $r < n + \sigma$, then $\gamma_K^{opt} = 0$ and $\gamma_H^{opt} = \beta/(1-\alpha)$.

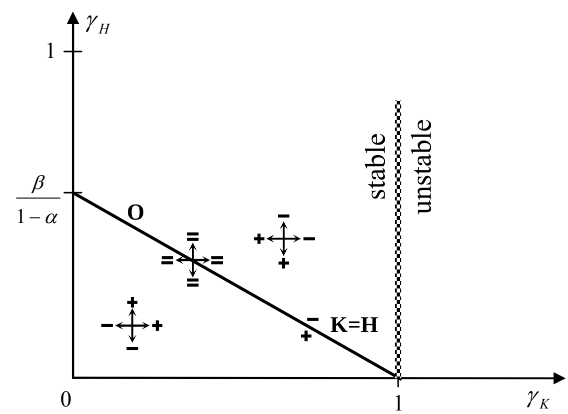
(c) If $r = n + \sigma$, then $\gamma_K^{opt} = 1 - \frac{1-\alpha}{\beta}\gamma_H$ and $0 < \gamma_H^{opt} < \frac{\beta}{(1-\alpha)}$.

Notice that in all three cases optimal investment rates satisfy the following equation:

$$(1-\alpha)\gamma_H^{opt} = \beta(1-\gamma_K^{opt}). \tag{24}$$

It is an equation of a straight line presented below.

Fig.4. The golden rule illustrated



It may be useful to provide some intuition behind this golden rule. Loosely speaking, in case (a) we need to be as far to the right on the line H as possible without breaching the stability condition. In case (b), we ought to be at the left end of the line H. In case (c) all points along the line H are equally good.

Obviously, in reality the real interest rate is variable – in certain periods it exceeds the natural rate of growth, in other periods it does not. Therefore, in real economies all three cases may happen, and there is no simple optimal strategy. In periods of cheap money it is worthwhile to invest heavily in human capital (at the cost of physical capital), but in periods of tight money this recipe (almost) turns around.

One thing is certain, however. According to (24), regardless of any (current or future, actual or expected) values of the real interest rate, the speed of population growth and the rate of technical progress, the economy should always stay on the line H defined by (24). A permanent increase in the cost of capital and/or a fall in the natural rate of growth should induce a movement along the line H towards its upper end, and vice versa.

6. NUMERICAL EXPERIMENTS

The baseline scenario

Let us assume the following base set of values of parameters:

$$\alpha = 1/3,\ \beta = 1/3,\ n = 0\%,\ \sigma = 3\%,\ \delta = 4\%,$$
$$r = 5\%,\ \gamma_K = 22\%,\ \gamma_H = 5\% \qquad (25)$$

We don't know what will be the values of parameters in Poland after the adoption of euro, so we should treat the above set of values and most of calculations below as an experiment (simulation) rather than a forecast[5].

In the baseline scenario, the stability boundary is equal to (approximately) 77,8%. We have case (a), since $r > n + \sigma$. The optimal rate γ_K^{opt} is infinitely close to the stability boundary, i.e. $\gamma_K^{opt} \cong 77{,}8\%$, whereas $\gamma_H^{opt} = \frac{\beta}{1-\alpha} \cdot \frac{r-n-\sigma}{r+\delta} \cong 11{,}1\%$. These values maximize the steady-state consumption, though it is clear that saving as much as 88,9% of national income would be very painful for current consumption (*more about it later*).

It's worth noticing that if the real interest rate falls below the level of $n + \sigma = 3\%$, then the optimal investment rates become completely different, because we switch to case (c). In this case the golden rule for the base set of parameters would yield: $\gamma_K^{opt} = 0\%$, and $\gamma_H^{opt} = \frac{\beta}{1-\alpha} = 50\%$.

Therefore it's virtually impossible to provide the exact recipe, how much should be saved and invested. The critical unknown is the level of the real interest rate. However, in our opinion it's possible to draw some enlightening conclusions regarding the level of investment in human capital, even though it's almost impossible to give responsible recommendation for investment in physical capital. Let us look now at some numerical experiments[6].

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[5] The rate of investment in human capital is very difficult to estimate. We set it at 5% of GDP, since this is the level of public expenditures on education in Poland. See e.g. Eurostat Yearbooks. The level of private spending on education is unknown.

[6] Calculations are performed for a discrete version of the model. We set the following initial values, for $t = 1$: $e(1) = -1$, $h(1) = 1$, $A(1) = 1$.

Scenario A. Permanent increase of γ_H by 2 p.p. (from 5% to 7%)

The graphs below illustrate the process of transitory dynamics caused by an increase of γ_H from 5% to 7%. In the first phase (for $1 \le t \le 100$) all parameter values are taken from the baseline scenario. In the second phase (starting from $t = 101$) the rate of investment in human capital is permanently higher (7% instead of 5%), whereas all other parameters are unchanged. The figures below illustrate the trajectories of selected variables. In the first phase the economy slowly converges towards the steady state determined by (25). The steadily growing stock of human capital attracts physical capital from abroad (see equation [k]), and we observe a permanent (though slow) increase in domestic output, national income and consumption.

Fig. 5. Investment in human capital (*iH*), and investment in physical capital (*i*)

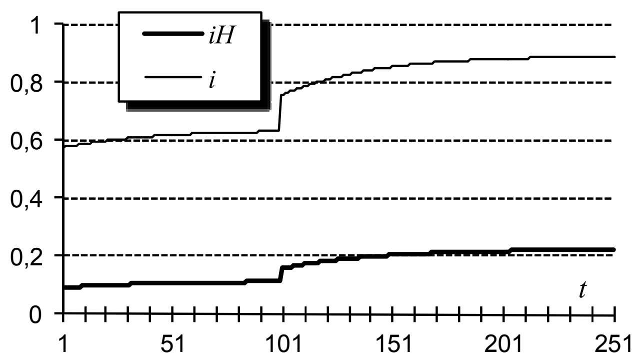


Fig. 6. The stock of human capital (*h*), and domestic output (*y*)

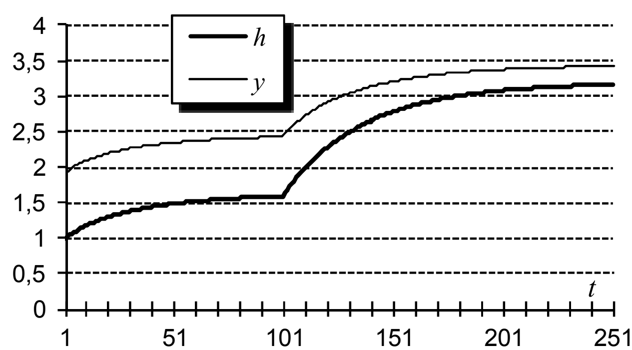
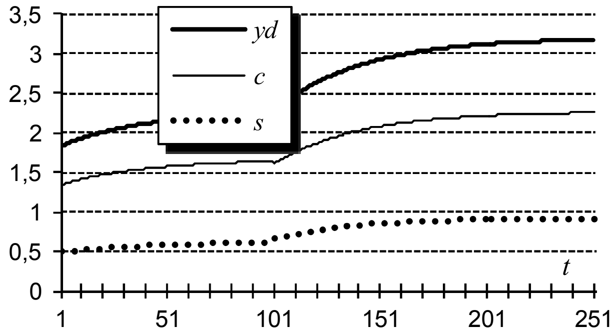


Fig. 7. Disposable income (*yd*), consumption (*c*), and savings (*s*)



In the second phase γ_H is equal to 7% instead of 5% of national income. Hence the new steady state appears, with much higher level of human capital. An increase in γ_H by 2 p.p. radically improves the steady-state level of human capital – it is almost twice as big as before (exactly 96% bigger). Of course, it takes many years until the economy approaches the new equilibrium. Nevertheless, thanks to improvement in human capital, the economy accelerates. Significant growth of human capital induces investment in physical capital (as well as attracts foreign investment), hence the stock of physical capital (k) grows significantly faster than in phase 1. The new steady-state level of domestic capital is 40% higher than in phase 1.

Fast growth of human capital coupled with significant inflow of foreign capital leads to a significant acceleration of production – the new-steady-state output is 40% higher than before. Last but not least, consumption in the new equilibrium is higher by as much as 36.2% ! That means a huge improvement in welfare, but we have to remember that the society may wait many years (or decades) to get close to the new steady state.

It's very important that the above percentage comparisons between old and new steady states are totally immune to changes in the level of the real interest rate, which is probably the most fluctuant parameter of the model. For example, if we set the real interest rate at 1% or 3% (or anything else) throughout the entire time horizon, holding all other parameters unchanged, the resulting percentage changes in the above simulation would be exactly the same. An increase of γ_H from 5% to 7% results in an increase of the steady-state consumption by 36.2% – regardless of the level of the real interest rate[7].

[7] To see why, divide two formulas for the steady-state consumption with two different rates of investment γ_H (only). It turns out that all terms including r cancel out. Hence, the resulting

Scenario B. Permanent increase of γ_K by 2 p.p. (from 22% to 24%)

The graphs below illustrate the process of transitory dynamics caused by an increase of γ_K from 22% to 24%. Similarly to scenario A, in the first phase (for $1 \le t \le 100$) all parameter values are taken from the baseline scenario. In the second phase (starting from $t = 101$) the rate of investment in physical capital is permanently higher (24% instead of 22%).

Fig. 8. Investment in human capital (*iH*), and investment in physical capital (*i*)

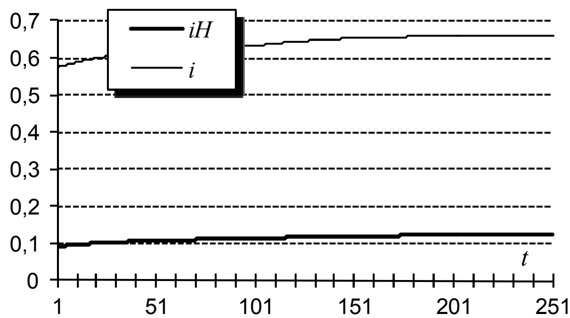
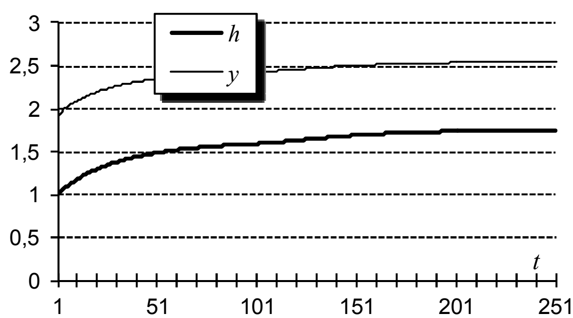
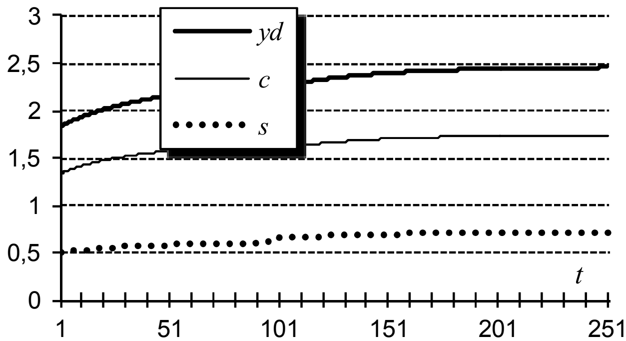


Fig. 9. The stock of human capital (*h*), and domestic output (*y*)



ratio only depends on the investment rates γ_K, γ_H, and the elasticities of the production function (α and β).

Fig. 10. Disposable income (*yd*), consumption (*c*), and savings (*s*)



The results of additional investment in physical capital are poor, if compared to scenario A. An increase in steady-state consumption amounts to a mere 4.6%, whereas the steady-state output increases by 3.7%, and human capital by 7.6%.

Similarly to scenario A, the above percentage comparisons between old and new steady states are immune to changes in the level of the real interest rate.

Scenario C. Permanent increase of γ_H (by 2 p.p.) at the cost of γ_K

Now, we keep the total rate of savings ($\gamma = \gamma_K + \gamma_H$) constant in both phases. However, the composition of national investment changes in the second phase. We assume that in the first phase (for $1 \le t \le 100$) $\gamma_K = 22\%$ and $\gamma_H = 5\%$ (just like in the baseline scenario). In the second phase $\gamma_K = 20\%$ and $\gamma_H = 7\%$, leaving the total rate of savings unchanged at 27%. The effects of this relatively small shift of resources from physical to human capital investment are surprisingly large. The steady-state stock of human capital grows by almost 82.7%, domestic capital (*k*) increases by 35.2%, the national income and consumption grow by 30.5%. The benefits are, therefore, almost as large as in scenario A, where the total rate of savings raised to 29%.

Transitory dynamics in this scenario is illustrated below.

Fig. 11. Investment in human capital (*iH*), and investment in physical capital (*i*)

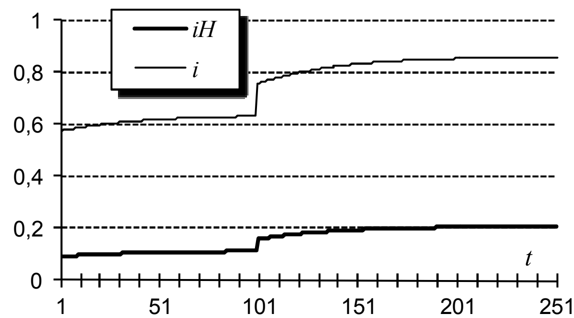


Fig. 12. The stock of human capital (*h*), and domestic output (*y*)

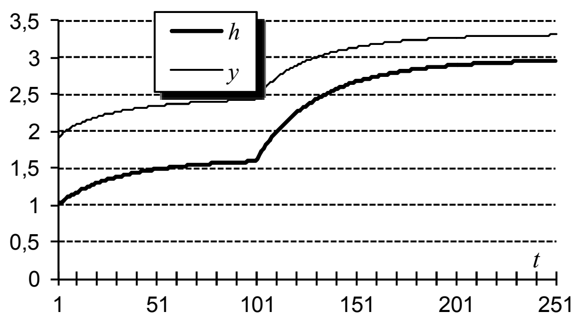
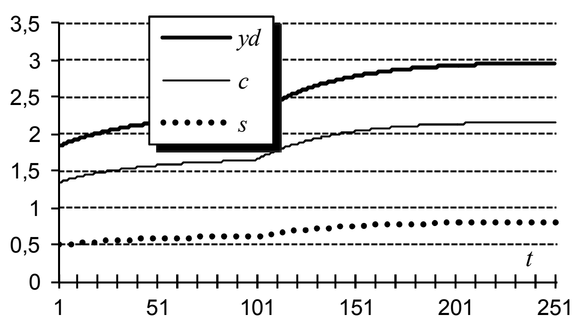


Fig. 13. Disposable income (*yd*), consumption (*c*), and savings (*s*)



The table below summarizes all of the above scenarios.

Table 2. The effects of changes in investment rates in three scenarios

	Base	**A**	**B**	**C**
$\gamma_K =$	22%	22%	24%	20%
$\gamma_H =$	5%	7%	5%	7%
human capital	100	196	108	183
physical capital	100	140	104	135
national income	100	140	108	130
consumption	**100**	**136**	**105**	**130**

Obviously, scenarios A and C are the best in terms of economic welfare. Clearly, even small (but permanent) increase in the rate of investment in human capital results in very large improvement in the steady-state income and consumption. If the country has limited resources (savings), and doubts whether to invest in physical capital or human capital, the answer is very clear. The best way to improve economic situation is investment in human capital. Of course, one has to remember that it may take decades before positive effects become significant and obvious to ordinary people.

6. MODIFICATIONS

Modification 1. What if human capital is mobile?

So far we assumed that human capital (as well as raw labor) is immobile. In this section we waive this assumption, allowing for certain level of human capital mobility. So far empirical research suggests that mobility of labor between EU countries is very low, probably because of cultural and language barriers. However, the most highly educated workers (various specialists, engineers, researchers etc.) are more mobile. We want to know how sensitive are the above simulations to human capital mobility.

Let us assume that the efficiency of investing in human capital is equal to 80%, (in other words 20% of invested resources is wasted, due to human capital outflow). Technically speaking, equation [*h*] will be replaced by its generalized version:

$$[h'] \quad \dot{h} = \omega\gamma_H y_d - \varphi h ,$$

where $\omega = 80\%$. It's easy to show that the golden rule remains unchanged, however the level of output, consumption etc. in the steady-state is decreasing with ω. The lower the level of efficiency of investment in human capital, the lower the steady state output. Mathematical details will be left for future analysis. Here we will only present simulation results.

The table below summarizes the effects of a simulation analysis. It contains percentage ratios obtained in scenarios A, B and C after introducing modification 1.

Table 3. Modification 1 results

	Base	A	B	C
$\gamma_K =$	22%	22%	24%	20%
$\gamma_H =$	5%	7%	5%	7%
human capital	100	196	108	183
physical capital	100	140	104	135
national income	100	140	108	130
consumption	**100**	**136**	**105**	**130**

The percentage ratios are exactly the same (it's not a copy-paste mistake)! They are completely insensitive to ω. Therefore, our conclusions remain unchanged. Scenarios A and C are the best. The best way to improve economic welfare is to invest in human capital (rather than in physical capital).

Although ω does not influence the percentage ratios, it DOES influence the levels of variables in the long-run equilibrium. For example, for $\omega = 100\%$, in the baseline scenario we have $c = 1.6689$, whereas for $\omega = 80\%$, $c = 1.3351$, which is exactly equal to 80% of 1.6689. Therefore the LEVEL of steady-state consumption is a linear function of ω. It's possible to prove that the steady-state value of consumption is equal to ω times the value for $\omega = 100\%$. It's clear, that with low value of ω (high outflow of human capital), the economy may fall into the classic poverty trap. The most highly educated people emigrate because the country is poor, and the country remains poor because of this emigration. It's a classic vicious circle.

Modification 2. The expanded definition of human capital

Some researchers define and measure human capital widely – including not only education, but also health statistics[8]. Therefore, we shall assume now that the actual level of investment in human capital is in reality higher than 5% assumed in our base set of parameters. It's almost impossible to measure it precisely, since it may include not only public but also private spending on

[8] See e.g. Sab and Smith (2001).

health care, plus health improvement (even spending on vacations may be treated as investment in health). Since we have no reliable data, we will simply set the value of γ_H at 15% (in the baseline scenario), and see how it changes the results of scenarios A, B and C, which are constructed analogously to the above experiment. In scenario A we increase γ_H by 2 p.p. In scenario B we do the same to γ_K. Finally, in scenario C we transfer 2 p.p. of national savings from investment in physical capital into investment in human capital.

Table 4. Modification 2 results

	Base	**A**	**B**	**C**
$\gamma_K =$	22%	22%	24%	20%
$\gamma_H =$	15%	17%	15%	17%
human capital	100	128	108	120
physical capital	100	113	104	109
national income	100	113	108	106
consumption	**100**	**110**	**104**	**106**

Once again, scenarios A and C are the best in terms of the steady-state consumption. However, the percentage ratios are far smaller than previously. It seems that our propositions are robust to the expanded definition of human capital. Even if we think about human capital in a wide sense, including education and health care, still investing in human capital is the best source of economic welfare.

Modification 3. Modified (Ramsey-style) optimization

The above golden rule was developed by solving a very simple optimization problem: the maximization of the steady-state consumption (per capita). It can be criticized on several grounds. First, in practice the convergence towards the steady state is usually a slow process (it may take decades to cover half of the distance). So why should we care about such a distant future. Secondly, the steady state is a moving target – due to changes in exogenous parameters and variables it constantly shifts. Therefore, skeptics could claim that it is an "academic exercise for algebra lovers" to analyze the properties of the steady state.

For the above reasons it is reasonable to modify the maximization criterion to include not only the steady-state consumption, but also the entire consumption stream during the transition period. Without mathematical rigor (*left for future analysis*), lets carry out an intellectual experiment – how will the golden rule change if we maximize the stream of consumption in the very long (perhaps even infinite) time horizon.[9] In case (c) it is no longer indifferent for consumers in what point on the line H the economy stays. Notice that the total rate of savings $\gamma = \gamma_K + \gamma_H$ rises when we move along line H towards its right end. Thus even though the steady-state consumption is equal along the entire line H, during the transition period the stream of consumption gets bigger when we move upwards along the line H. Therefore, in case (c) the optimal point would be at the upper left end of line H, i.e. $\gamma_K^{opt} = 0$ and $\gamma_H^{opt} = \beta/(1-\alpha)$.

Obviously, in case (b) the golden rule would remain unchanged, because the upper end of line H not only maximizes the steady-state consumption – it also maximizes the stream of consumption during the transition period. The situation is not so trivial in case (a). We have concluded that if $r > n + \sigma$ then the optimum lies at the right end of line H, where the total savings are the biggest, i.e. consumers give up a large part of their current consumption in order to maximize the steady-state consumption. Taking into account the stream of consumption during the transition period obviously weakens our conclusion. The real interest rate has to be significantly higher than the natural rate of growth in order to persuade consumers to give up current consumption in exchange for future (steady-state) consumption.

To summarize, a wider optimization criterion, which incorporates the entire stream of consumption, results in the following golden rule: $\gamma_K^{opt} = 0$, and $\gamma_H^{opt} = \beta/(1-\alpha)$, unless the real interest rate is significantly[10] higher than natural rate of growth – in this case it is worthy to invest in physical capital as much as possible (without breaching the stability condition), along the line H, i.e. $\gamma_K^{opt} \to \frac{\varphi}{r+\delta}$ and $\gamma_H^{opt} \to \frac{\beta}{(1-\alpha)} \frac{r-n-\sigma}{r+\delta}$. Importantly, the optimal investment rates always satisfy equation (25), regardless of the actual form of consumer preferences (with respect to current and future consumption), and regardless of current and future (expected) values of other parameters.

SUMMARY

A small economy in an EMU has no better strategy of promoting economic growth than investing in human capital. If for some reasons the total amount of savings (that can be devoted to investment) is limited, and the economy has to chose between investing in human capital vs. physical capital, the choice is

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[9] Formally, there are several possible criteria, e.g. the (discounted) sum (integral) of consumption or a (discounted) sum (integral) of utility derived from consumption.

[10] How large – it depends on consumer preferences regarding current and future consumption.

clear. We demonstrate that resources invested in human capital can bring benefits which far exceed those provided by investing in physical capital. Importantly, these conclusions are immune to changes in the level of real interest rates. However, human capital mobility may be a serious problem. If the economy suffers from significant outflow of the top-quality human capital, it may fall into a genuine poverty trap.

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ABSTRACTS

Modelling Economies in Transition 2011
Łódź 2012, p. 141

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Almost Periodically Correlated Time Series in Business Fluctuations Analysis

Abstract. We propose a non-standard subsampling procedure in order to make formal statistical inference about the business cycle, one of the most important unobserved feature characterising fluctuations of economic growth. We show that some characteristics of business cycle can be modelled in a non-parametric way by discreet spectrum of the Almost Periodically Correlated (APC) time series. On the basis of estimated characteristics of this spectrum it is possible to extract business cycles by filtering. On the basis of our results we characterise the main properties of business cycles in industrial production index for Polish economy.

Modelling Economies in Transition 2011
Łódź 2012, pp. 142–143

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Transitional Dynamics and Business Cycle Phases in Poland

Abstract. Hamilton's (1989) Markov switching model assumes the transition probabilities between business cycle phases and expected duration of each phase are constant over time. Filardo (1994) extended Hamilton's model by time-varying state transition probabilities. He allowed these probabilities to evolve as logistic functions of observable economic fundamentals. Durland, McCurdy (1994) assumed the transition probabilities can be duration dependent.

The aim of the analysis is modeling of business cycle in Poland since the beginning of 1995 to September 2011 on the basis of quarterly real GDP and monthly production in manufacturing using the time-varying transition probabilities (TVTP) Markov switching model. It was checked if the state transition probabilities vary over time together with the evolution of the Composite Leading Indicator published by OECD and if business cycle phases in Poland are duration dependent.

As a benchmark there were firstly estimated Markov switching autoregressive models with fixed transition probabilities (FTP) with two states: upturn and downturn of economic growth. The best performance was revealed for FTP AR(1) and AR(2) on the basis of first differences of natural logarithm for real GDP and production in manufacturing respectively. Afterward the business cycle dating was conducted. The moment in time was assigned to the slowdown phase if the probability of being in a slowdown was greater than 0.1. Otherwise was assigned to be in expansion. More over it was assumed that every cycle phase should last more than two quarters.

On the basis of these assumptions the duration variable was constructed and TVTP model was estimated. The duration variable took for example the value 1 if the peak/trough was last period, while if the peak/trough was 5 periods berofe, the duration variable would take the

value 5. To improve estimation the values of the duration variable were arbitrarily restricted to some maximum value D*. The likelihood–ratio test with the null hypothesis of no time variation in the transition probabilities did not accept the FTP model for both GPD and production in manufacturing, but the parameters by the duration variable were statistically insignificant. It means that both for GDP and production in manufacturing the phase duration is not important in predicting the end of downturn and upturn either.

The TVTP model was also estimated under the assumption that the state transition probabilities change over time together with the evolution of economic fundamentals represented by the composite leading indicator. The likelihood-ratio test supported the importance of time-varying state transition probabilities for GDP and production in manufacturing. The signs of the two coefficients of interest should be opposite (negative for downturn and positive for upturn), only then the interpretation that the state transition probabilities change together with fluctuations of the composite leading indicator is obvious. For example when the leading indicator grows the probability of being in the upturn increases. For GDP the coefficients were not of opposite signs, which made the results difficult to interpret. In the TVTP model for production in manufacturing the coefficient had an opposite sign, but the parameter for downturn was not significant.

Modelling Economies in Transition 2011
Łódź 2012, p. 144

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The European CO_2 Emissions Trading System (EU-ETS): The Good, the Bad and the Interesting*

Abstract. In January 2005 the EU-wide CO_2 emissions trading system (EU-ETS) has formally entered into operation. Within the new trading system, the right to emit a particular amount of CO_2 has become a tradable commodity, called EU Allowance (EUA), and carbon pricing has become an important mechanism for providing companies with incentives to invest in carbon abatement. However, price formation in carbon markets involves a complex interplay between policy targets, dynamic technology costs and market rules.

In this paper we review the basic characteristics of carbon markets and investigate the relationship between spot and futures prices within the EU-ETS. We conduct an empirical study on price behavior, volatility term structure and correlations in different CO2 EUA contracts during the pilot trading and Kyoto commitment periods. We find that while for the pilot trading period (2005–2007) the market was initially in backwardation, after the news of overallocation, both allowance prices and convenience yield approached zero. During the Kyoto commitment period (2008–2012), the market has changed from initial backwardation to contango with significant convenience yields in futures contracts.

We further examine the dynamic structure of the relationship between spot and futures prices in the functional form by applying a relatively new approach of dynamic semiparametric factor models (DSFM). Interestingly, our DSFM results can be related to the classic Gibson-Schwartz two-factor model for pricing contingent claims in commodity markets that uses the spot price and the instantaneous convenience yield as factors. Our results might point towards future applications of the Gibson-Schwartz model for pricing of intra- and inter-period EUA derivatives contracts.

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* In collaboration with Stefan Trueck (Department of Applied Finance and Actuarial Studies, Macquarie University, Sydney, Australia).

FINANCIAL CRISIS

Modelling Economies in Transition 2011
Łódź 2012

ABSTRACTS

Modelling Economies in Transition 2011
Łódź 2012, pp. 149

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Linkages in Global Stock Market During the Recent Crisis: A Comparison of Acute and Creeping Phases

Abstract: The aim of the paper is to investigate the dynamics of linkages between stock markets during the recent financial crisis. We are interested in similarities and differences between the patterns of changes in the conditional dependence structure during distinct phases of the crisis. The basic tool in the performed analysis is a Markov-switching copula model applied to pairs of the daily returns on selected representative stock indices. The model enables us to distinguish regimes without extreme dependence, and ones with tail dependence which can be of asymmetric type. We are thus able to examine the linkages between chosen stock markets, focusing on a comparison of the strength of the tail dependencies during the considered periods.

Modelling Economies in Transition 2011
Łódź 2012, p. 150

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Dynamics of the sovereign Credit Default Swaps and the evolution of the financial crisis in the Central Europe

Abstract. The aim of the presentation is to analyse the dynamics of the sovereign Credit Default Swaps (sCDS) instruments in the context of the financial crisis. The stress is put on the Central Europe. The author analyses the regional dependencies and verifies the influence of the volatility of the rest of the European instruments on the volatility of the Central European ones.

Modelling Economies in Transition 2011
Łódź 2012, p. 151

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The Impact of the World Financial Crisis on the Polish Interbank Market: a Swap Spread Approach

Abstract. The swap spread is defined as the difference between the fixed-rate of an interest rate swap and the yield of the treasury with the same maturity. Huang and Neftci (2003) presented the following interpretation of the swap spread: the effective proxy of bank liquidity and the credit spread indicator. Swap spread reacts also to the government efforts to manage the national debt. Swap spread is strongly correlated with credit spread, which is interpreted as credit risk premium. The main goal of my research is understanding changes in the Polish interbank market, which arose after the transmission of the world financial crisis to Poland. This analysis is based on properties of the swap spreads determined on the basis of Polish zloty interest rate swaps and Polish Treasury benchmarks with 1, 2 and 5- year maturities. Following Castagnetti (2004) I place big emphasis on determining the relations between the conditional mean and the conditional variance.

Modelling Economies in Transition 2011
Łódź 2012, p. 152

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Excessive Imbalance Procedure in the EU: a welfare evaluation

Abstract. I develop a framework for assessing the welfare implications of the new EU's Excessive Imbalance Procedure (EIP) to be implemented in 2012, with a special focus on the current account constraint. For this purpose, I apply a New Keynesian 2-region, 2-sector DSGE model, using the second order Taylor approximation of the households' utility around the steady state as a standard measure of welfare. The compliance with the CA criterion is ensured by modifying the policymakers' loss function in line with Woodford's (2003) treatment of the zero lower bound of nominal interest rates. I also compare the impact of the EIP between countries along the dimensions of catching-up process and Euro area membership.