Lodz
Economics
Working
Papers

Measuring Uncertainty of Optimal Simple Monetary Policy Rules in DSGE Models

Mariusz Górajski,
Zbigniew Kuchta

6/2018

FACULTY OF ECONOMICS AND SOCIOLOGY
University of LODZ

Measuring uncertainty of optimal simple monetary policy rules in DSGE models[1]

Mariusz Górajski[2], Zbigniew Kuchta[3]

First draft: 22 July 2018

This draft: 24 September 2018

Abstract

This paper presents a new approach to measure the parameter uncertainty for optimal simple monetary policy rules in the New Keynesian dynamic stochastic general equilibrium models. More precisely, we propose a new algorithm which enables to directly introduce parameter uncertainty into the optimal simple precommitment rule problem. As a result we find distributions of the optimal monetary policy reactions and the minimized welfare losses. To compare the distributions of the monetary policy parameters and the welfare losses we apply the first order stochastic dominance ordering (SD1). The SD1 inequality between the probability distribution is verified by means of the Kolmogorov-Smirnov test. The proposed algorithms are applied to the Erceg, Henderson and Levine (2000) small-scale closed economy model estimated for the Polish economy. For the welfare-loss-minimizing central bank, we examine three types of the dynamic specification of its policy rule: backward-, current- and forward-looking. Finally, for a given set of optimal and implementable monetary policy rules, we show that the fully specified forward-looking monetary policy rule with interest rate smoothing mechanism minimizes the welfare-loss in the sense of the stochastic ordering SD1.

Keywords: optimal monetary policy, DSGE, uncertainty.

JEL codes: E47, E52

---

[1]This work was supported by the National Science Centre in Poland under Grant No. 2017/26/D/HS4/00942.
[2] Department of Econometrics, Faculty of Economics and Sociology, University of Lodz, Poland, e-mail mariusz.gorajski@uni.lodz.pl.
[3]Department of Economic Mechanisms, Faculty of Economics and Sociology, University of Lodz, Poland, email zbigniew.kuchta@uni.lodz.pl.

1. Introduction

A generally accepted principle of economics states that policymakers in central banks should act in an optimal manner ((see (Tinbergen 1952), (Blinder 1999), (Clarida, Gali, and Gertler 1999), (Taylor 1999), (Woodford 2003))). When making policy decision agents face a problem of constrained optimization, where the goal of an optimal central bank is to choose the policy instrument to minimize the expected welfare loss subject to dynamic macroeconomic equations that contain forward-looking rational expectations. There are several sources of uncertainty that can disturb the monetary policy rules (see (Poole 1998), (Goodhart 1999), (Blinder 1999), (Greenspan 2004), (Onatski and Williams 2003); (Woodford 2003)). The main source of this randomness corresponds to exogenous shocks disturbing the macroeconomic variables from their steady-state values. The Bayesian approach to macroeconomic modeling assumes that the posterior distribution of model parameters is another source of uncertainty. Many researchers and central bank practitioners emphasize that due to uncertainty a little stodginess of the central bank policy-makers is entirely appropriate (see Blinder 1999). Moreover, (Chow et al. 1975) reported that there is no clear dependence between the parameter uncertainty and the policy rules. Thus quantitative research on the impact of uncertainty on the shape of optimal macroeconomic policies is required.

The usual approach to optimal monetary policy implementation in dynamic stochastic general equilibrium models (DSGE models) assumes that when solving the optimal control problem of minimizing welfare losses the fixed values of structural model parameters are taken into account with certainty (see (Erceg, Henderson and Levine 2000), (Giannoni and Woodford 2002), (Taylor and Williams 2010) and references therein).

When the Knightian robust policy rules are used, the distributions of parameter uncertainty are not available. In the first step central bank policymakers consider the worst-case scenario by maximizing the welfare loss over the range of plausible parameter values and, then in the second stage, she minimizes this worst-case value of welfare loss with respect to policy instruments (cf. (Kendrick 2005), (Onatski and Williams 2003)). In several research papers, the authors using the min-max technique proved that the robust optimal policy rule gives more aggressive responses of the interest rate to inflation and the output gap shocks than is the case of parameter certainty (cf. (Onatski and Stock 2002), (Giannoni 2002) and (Giannoni 2007)). There is also possible to construct mean robust monetary policy rule for models with parameter uncertainty (see (Justiniano and Preston 2010), (Górajski 2017)).

Using this approach we minimize the expected value of welfare loss, where the expectation is also taken with respect to the random model's parameters. In these approaches, both robust policy reaction functions are derived with certainty. Moreover, robust Bayesian rules are also designed to account for both model and parameter uncertainty (see Levine et al. (2012), Cogley et al. (2011) and reference therein). In all above approaches, we observe the following inconsistency between simple optimal, robust policy rules and the macroeconomic model, the optimal and robust response coefficients are given by the unique deterministic numbers, while the assumed macroeconomic model is observed with parameter uncertainty. As optimal or robust central banks act in the uncertain environment they should not be the prisoners of a single vector of response coefficients. To solve this inconsistency we assume that the optimal policy coefficients are random variables with probability distributions inherited from the posterior distributions of structural model parameters.

Our paper makes two principal contributions. First, we propose a new algorithm which enables us to find and examine posterior distributions of the optimal monetary policy reactions and the minimized welfare losses. To compare the distributions of the monetary policy parameters and the welfare-losses we apply the first order stochastic dominance ordering (SD1). The SD1 inequality between the probability distribution is verified by means of the Kolmogorov-Smirnov test. The second contribution involves the application of our approach to set of optimal and implementable monetary policy rules. The proposed algorithms are applied to the Erceg, Henderson and Levine (2000) small-scale closed economy model estimated for the Polish economy. For the welfare-loss-minimizing central bank, we examine three types of the dynamic specification of its policy rule: backward-, current- and forward-looking. Finally, for a given set of optimal and implementable monetary policy rules, we show that the fully specified forward-looking monetary policy rule with interest rate smoothing mechanism minimizes the welfare-loss in the sense of the stochastic ordering SD1.

The paper is organized as follows. In the next section, we introduce the Erceg, Henderson and Levine small-scale DSGE model. In Section 3 we present the Bayesian estimation procedure for DSGE models**.** Section 4 contains our new algorithm to measure the uncertainty of optimal policy reactions. In Section 5 we perform uncertainty assessment of optimal monetary policy in small-scale DSGE model estimated for the Polish economy. In the last section, we conclude our findings.

2. The theoretical model

This section considers a closed economy version[4] of the New Keynesian model (Erceg, et. al. 2000) with sticky prices and wages á la Calvo (1983). Our economy consists of final good firms, the intermediate good firms, the labor agency, and households. It is assumed that firms are indexed by $j \in [0;1]$, whereas households by $i \in [0;1]$.

Final good, Y_t , consists with an infinite number of non-perfectly substitutive intermediate goods, $Y_t(j)$, and it is produced according to following the technology (Dixit, Stiglitz, 1977):

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{1}{1+\tau_p}} dj\right]^{1+\tau_p} \tag{1}$$

where: $\tau_p > 0$ is the monopolistic mark-up on the goods market. Representative final good firm maximizes its profits and treats the price of a final good P_t and price of intermediate *j*-good $P_t(j)$ as given. Thus, the optimal demand function is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\tau_p}{\tau_p}} Y_t \tag{2}$$

for all $j \in [0;1]$ and where the implied general level of prices is given by:

$$P_t = \left[\int_0^1 P_t(j)^{-\frac{1}{\tau_p}} dj\right]^{-\tau_p} \tag{3}$$

We assume that every intermediate good *j* is produced by an monopolistically competitive firm using only labor inputs L_t^j according to the following technology:

$$Y_t(j) = \varepsilon_t^a L_t^j \tag{4}$$

[4] Although Poland is an example of the small scale open economy we focus on closed economy model since it allows us to limit number of parameters and consequently limits the computational effort. It is worth noting that Krajewski (2015), Baranowski and Kuchta (2015) as well as Baranowski et. al. (2016) estimate or calibrate the closed economy DSGE model for Poland.

where: ε_t^a represents the level of technology which evolves according to stationary AR(1) process:

$$\ln \varepsilon_t^a = (1-\rho_a)\ln \varepsilon^a + \rho_a \ln \varepsilon_{t-1}^a + \sigma_a \eta_t^a;\ \ \eta_t^a \sim i.i.d.N(0;\ 1) \tag{5}$$

where: $\rho_a \in (0;1)$ is an autoregressive parameter and $\sigma_a > 0$ represents the standard deviation of technological shock.

We assume that each firm hires labor at a perfectly competitive labor market and pays a real wage, w_t. Under the technology of production (4) the real marginal cost $RMC_t(j)$ does not depend on the level of output:

$$RMC_t(j) = \frac{w_t}{\varepsilon_t^a} \tag{6}$$

Following Calvo (1983) and Yun (1996), it is assumed that in each period only a randomly chosen part of intermediate firms, $1-\theta_p \in (0;1)$, is able to reoptimize its price. Each firm chooses the price to maximize the expected sum of discounted profits:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_p)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}(j)\left[\frac{P_t(j)}{P_{t+s}} - RMC_{t+s}(j)\right]\right\} \tag{7}$$

subject to demand function (2), where $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor and E_t is the rational expectation operator. The rest of the prices remains constant. The first order condition is given by:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\theta_p)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}^*(j)\left[(1+\tau_p)RMC_{t+s}(j) - \frac{P_t^*(j)}{P_{t+s}}\right]\right\} = 0 \tag{8}$$

where:

$$Y_{t+s}^*(j) = \left(\frac{P_t^*(j)}{P_{t+s}}\right)^{-\frac{1+\tau_p}{\tau_p}} Y_{t+s} \tag{9}$$

Condition (8) shows that the intermediate good firm chooses the price to equate expected average future marginal revenues to average future expected markups over real marginal cost (Schmitt-Grohe, Uribe, 2004a, p. 13). Since all reoptimizing firms face identical demand curve (2) and real marginal cost (2), they will choose the same price. This property allows us to express the price of a final good (3) as:

$$P_t = \left[(1-\theta_p) P_t^{*\,-\frac{1}{\tau_p}} + \theta_p P_{t-1}^{-\frac{1}{\tau_p}} \right]^{-\tau_p} \tag{10}$$

Labor services, L_t, are provided by an agency which aggregate the heterogeneous labor services, $L_t(i)$, delivered by households, into homogenous input using the following technology:

$$L_t = \left[\int_0^1 L_t(i)^{\frac{1}{1+\tau_w}} di \right]^{1+\tau_w} \tag{11}$$

where: $\tau_w > 0$ is the monopolistic mark-up on the labor market. The optimal demand for labor is represented by:

$$L_t(i) = \left(\frac{w_t(i)}{w_t} \right)^{-\frac{1+\tau_w}{\tau_w}} L_t \tag{12}$$

for all $i \in [0; 1]$ and where $w_t(i)$ is the real wage of household i and the real wage w_t is given by:

$$w_t = \left[\int_0^1 w_t(i)^{-\frac{1}{\tau_w}} di \right]^{-\tau_w} \tag{13}$$

Each household tends to maximize the lifetime utility described by:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \varepsilon_{t+k}^b \left[\frac{C_{t+k}(i)^{1-\delta_c}}{1-\delta_c} - \frac{L_{t+k}(i)^{1+\delta_l}}{1+\delta_l} \right] \right\} \tag{14}$$

where: $C_t(i)$ is consumption, $\beta \in (0; 1)$ is a subjective discount factor, $\delta_c > 0$ represents the relative risk aversion parameter, $\delta_l > 0$ is the inverse of Frish's labor elasticity, ε_t^b denotes preference shock which follows a stationary AR(1) process:

$$\ln \varepsilon_t^b = (1-\rho_b) \ln \varepsilon^b + \rho_b \ln \varepsilon_{t-1}^b + \sigma_b \eta_t^b;\; \eta_t^b \sim i.i.d. N(0;\, 1) \tag{15}$$

where: $\rho_b \in (0;1)$ is autoregressive parameter and σ_b is the standard deviation of preference shock.

It is assumed that each household has access to the market of nominal bonds, $B_t(i)$, participates in state-contingent securities system which prevent from idiosyncratic risks connected with wage rigidities. It also it receives income from shares of intermediate goods firms, $A_t(i)$.[5] Thus, the intertemporal household's budget constraint is given by:

$$\frac{B_t(i)}{P_t R_t} + C_t(i) = \frac{B_{t-1}(i)}{P_t} + w_t(i)L_t(i) + A_t(i) \tag{16}$$

where: R_t is the nominal interest rate.

The maximization of lifetime utility function (14) subject to a set of intertemporal budget constraints (16), results in the following Euler equation:

$$\frac{\varepsilon_t^b}{C_t(i)^{\delta_c}} = \beta E_t \left\{ \frac{\varepsilon_{t+1}^b}{C_{t+1}(i)^{\delta_c}} \frac{R_t}{\pi_{t+1}} \right\} \tag{17}$$

Under the transversality condition: :

$$\lim_{t\to\infty} \beta^t \frac{\varepsilon_t^b}{C_t(i)^{\delta_c}} B_t(i) = 0 \tag{18}$$

and where: $\pi_t = \frac{P_t}{P_{t-1}}$ is the inflation rate.

Similarly to intermediate firm's problem, each household chooses its wage, $w_t(i)$, according to Calvo scheme (see. Schmitt-Grohe, Uribe, 2005). In every period only a randomly chosen and constant part of household, $1-\theta_w \in (0;1)$, can reoptimize its wage. It maximizes the lifetime utility, given by:

$$E_t \left\{ \sum_{k=0}^{\infty} \beta^k \theta_w^k \varepsilon_{t+k}^b \left[\frac{C_{t+k}(i)^{1-\delta_c}}{1-\delta_c} - \frac{L_{t+k}(i)^{1+\delta_l}}{1+\delta_l} \right] \right\} \tag{19}$$

subject to budget constraint (16) and labor demand (12). The first order condition is given by:

[5] State-contingent securities system prevents household from idiosyncratic risk arised from staggered wage setting. It is assumed that payments from this systems eliminate income inequalities between households in a given period.

$$E_t\left\{\sum_{k=0}^{\infty}(\beta\theta_w)^k L_{t+k}(i)^*\left[(1+\tau_w)MUL_{t+k}(i)^* - \frac{MUC_{t+k}(i)}{\prod_{m=1}^{k}\pi_{t+m}}w_t(i)^*\right]\right\} \tag{20}$$

where: $MUL_t(i)^*$ is the optimal level of marginal disutility of labor , $MUC_t(i)$ is marginal utility of consumption and

$$L_{t+k}(i)^* = \left(\frac{w_t(i)^*}{w_{t+k}}\right)^{-\frac{1+\tau_w}{\tau_w}} L_{t+k} \tag{21}$$

where: $w_t(i)^*$ is the optimal real wage.

Condition (20) indicates that the household chooses the optimal wage to equalize the average expected markup over the real marginal cost of working with the average expected marginal benefit of working, both expressed in utility terms (see Schmitt-Grohe, Uribe, 2004). Moreover, under the assumption of symmetric equilibrium, all households that choose wage in a given period those the same real wage. The rest of wages, namely $\theta_w \in (0;1)$ remains constant. Introduction of sticky wages causes that the dynamics of real wage (13) is expressed by:

$$w_t = \left[\theta_w\left(\frac{w_{t-1}}{\pi_t}\right)^{-\frac{1}{\tau_w}} + (1-\theta_w){w_t^*}^{-\frac{1}{\tau_w}}\right]^{-\tau_w} \tag{22}$$

where: w_t^* is the optimal real wage under symmetric equilibrium.

Finally, we impose the following equilibrium conditions on labor and goods market:

$$\frac{1}{\Delta_t(p)}\int_0^1 Y_t(j)dj = Y_t \tag{23}$$

$$\frac{1}{\Delta_t(w)}\int_0^1 L_t(i)di = L_t \tag{24}$$

and the aggregate demand equation:

$$Y_t = C_t \tag{25}$$

where: $\Delta_t(p) = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{1+\tau_p}{\tau_p}} dj \geq 1$ and $\Delta_t(w) = \int_0^1 \left(\frac{w_t(i)}{w_t}\right)^{-\frac{1+\tau_w}{\tau_w}} dj \geq 1$ are inefficient price and wage dispersions, respectively.

Alternative monetary policy rules

The presented model should be closed by some rule setting the level of interest rate. Traditionally, the New Keynesian DSGE models use a Taylor-type rule which links the interest rate with endogenous variables. Although the original Taylor's rule (1993) sets the federal fund rate as a function of inflation over the previous year and the output gap, it was extensively modified, obtaining vastness of different forms. Including all possible forms seems to be a tremendous task. Hence, the set of interest rate rule is limited to following "general" specification of the Taylor rule, including only measurable variables from the theoretical model:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{Y_{t+i}}{Y}\right)^{\phi_y}\right\} E_t\left\{\left(\frac{w_{t+i}}{w}\right)^{\phi_w}\right\} \quad (26)$$

for $i = \{-1;\ 0;\ 1\}$ and where: variables without time subscript denote steady state values, $\rho_r \in (0; 1)$ is the interest rate smoothing parameter, $\phi_\pi > 0$,$\phi_y > 0$and $\phi_w > 0$are inflation, output and real wage reaction parameters, respectively.

Table1. Specifications of simple monetary policy rules.

Rule number	Functional form
1	$\frac{R_t}{R} = E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\}$
2	$\frac{R_t}{R} = E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{w_{t+i}}{w}\right)^{\phi_w}\right\}$
3	$\frac{R_t}{R} = E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{Y_{t+i}}{Y}\right)^{\phi_y}\right\}$
4	$\frac{R_t}{R} = E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{Y_{t+i}}{Y}\right)^{\phi_y}\right\} E_t\left\{\left(\frac{w_{t+i}}{w}\right)^{\phi_w}\right\}$
5	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\}$
6	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{w_{t+i}}{w}\right)^{\phi_w}\right\}$
7	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{Y_{t+i}}{Y}\right)^{\phi_y}\right\}$
8	$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} E_t\left\{\left(\frac{\pi_{t+i}}{\pi}\right)^{1+\phi_\pi}\right\} E_t\left\{\left(\frac{Y_{t+i}}{Y}\right)^{\phi_y}\right\} E_t\left\{\left(\frac{w_{t+i}}{w}\right)^{\phi_w}\right\}$

Our interest rate rule (26) includes interest rate smoothing and reaction to inflation, output and real wage. Although the first three components are quite standard and well-motivated, the last one may put some doubts. We include real wage to emphasize the role of an alternative measure of economic activity as well as a variable which more direct link to the labor market [6]. Moreover, we allow for 0$^{\text{th}}$ restrictions on parameters $\rho, \phi_\pi, \phi_y, \phi_w$. As a results, we obtain 8 different specification of the Taylor rule. Table 1 sums up notation of different rules regarding to the 0-th restrictions on some parameters.

Finally, we consider 3 different dynamic specifications for each rule: backward-looking $(i = -1)$, current-looking $(i = 0)$ and forward-looking $(i = 1)$. Hence we work with 24 New Keynesian models differing by monetary policy rule[7].

3. Bayesian estimation of DSGE models

The New Keynesian models are estimated using Bayesian techniques[8] according to the Bayes theorem:

[6] Although the National Central Bank of Poland acts according to direct inflation targeting, the law allows to target other variables, if it is consistent with stabilization of prices.
[7] For simplicity we omit all mixed dynamic specifications of monetary policy rules as well as measures of dynamics for particular reactions variables.

$$p(\boldsymbol{\theta}, \boldsymbol{\phi}, |\mathbf{Y_T}; \boldsymbol{\omega}) = \frac{p(\boldsymbol{\theta}, \boldsymbol{\phi})\mathcal{L}(\mathbf{Y_T}|\boldsymbol{\theta}, \boldsymbol{\phi};\ \boldsymbol{\omega})}{p(\mathbf{Y_T})} \qquad (27)$$

where: $\mathcal{L}(\mathbf{Y_T}|\boldsymbol{\theta}, \boldsymbol{\phi};\ \boldsymbol{\omega})$ is a likelihood function, $p(\boldsymbol{\theta}, \boldsymbol{\phi})$ is a prior distribution, $p(\boldsymbol{\theta}, \boldsymbol{\phi}, |\mathbf{Y_T})$ is posterior distribution, $p(\mathbf{Y_T})$ represents the marginal data density, $\boldsymbol{\theta} = [\delta_c, \theta_p, \theta_w, \rho_a, \rho_b, \sigma_a, \sigma_b, \sigma_{m_r}, \sigma_{m_w}]'$ is a vector containing structural non-policy estimated parameters, $\boldsymbol{\omega} = [\delta_l, \tau_w, \beta]'$ is a vector of structural non-policy calibrated parameters [9] and$\boldsymbol{\phi} = [\rho, \phi_\pi, \phi_y, \phi_w]'$represents vector of monetary policy rule parameters[10].The Bayesian approach emphasizes the role of uncertainty about model and parameters, treating the latter as a random variables. According to likelihood principle, posteriors contain all relevant information about parameters obtained from the data $\mathbf{Y_T}$, including accuracy of estimates. We refer to likelihood principle by treating posteriors as a measure of parameter and model uncertainty.

The Bayesian estimation of DSGE models is performed in several steps[11]. In the first step, each equation is log-linearized around the steady state. As a result, we obtain the rational expectations system of equations:

$$\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})E_t\{\mathbf{x_{t+1}}\} + \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{x_t} + \mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{x_{t-1}} + \mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\boldsymbol{\varepsilon}_\mathbf{t} = \mathbf{0} \qquad (28)$$

where: $\mathbf{x_t} = [\hat{y}_t, \hat{r}_t, \hat{\pi}_t, r\widehat{mc}_t, \widehat{mrs}_t, \widehat{w}_t, \hat{\varepsilon}_t^b, \hat{\varepsilon}_t^a]'$ is a vector of endogenous variables[12], $\boldsymbol{\varepsilon}_\mathbf{t} = [\eta_t^a, \eta_t^b]'$ is a vector of innovations in technology and preference shocks, respectively, and $\mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega}), \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega}), \mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega}), \mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})$ are matrices which elements are functions of $\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega}$.

Next, the system (28) is solved by applying a perturbation method with first-order approximation for the policy and transition function (Schmitt-Grohe, Uribe, 2004b). The solution determines the transition equation in the state space representation of the DSGE model and has the form of:

---

[8] Alternatively, DSGE models can be estimated using the generalized method of moments (Christiano, Eichenbaum, 1992), the simulated method of moments (Duffie, Singleton, 1993), the indirect inference (Smith, 1993) or the maximum likelihood (Altug, 1989).

[9] Although the vector $\boldsymbol{\omega}$ may be included into vector $\boldsymbol{\theta}$, our notation distinguishes them and emphasizes the common empirical strategy of calibrating some non-identified or poorly identified parameters in applied works (eg. Smets, Wouters, 2003; 2007).

[10] Vector $\boldsymbol{\phi}$ differs among particular models.

[11] It is worth noting that we estimate each model separately.

[12] The variable $\hat{x} = \ln\left(\frac{x_t}{x}\right)$ denotes the percent deviation from steady state.

$$\mathbf{x_t} = \mathbf{F}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{x_{t-1}} + \mathbf{G}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\boldsymbol{\varepsilon}_\mathbf{t} \tag{29}$$

The state space model is closed by the measurement equation of the form:

$$\mathbf{x_t} = \mathbf{H}\mathbf{y_t} + \mathbf{v_t} \tag{30}$$

where: $\mathbf{y_t} = [\hat{y}_t^{obs}, \hat{r}_t^{obs}, \hat{\pi}_t^{obs}, \hat{w}_t^{obs}]'$ is a vector of observables, $\mathbf{H}$ is a matrix which links observables with their model's counterpart and $\mathbf{v_t}$ is a vector of measurement errors. The measurement errors should be added to omit the singularity problem. The theoretical model includes two structural shocks whereas four variables are observed, hence there is a need to add two measurement errors. It is assumed that nominal interest rate and real wage are measured with errors, represented by $\eta_t^{m_r}$ and $\eta_t^{m_w}$, respectively. As a result, we obtain the linear system of equations (29) – (30) with normal distributions of innovations and measurement errors. Hence, the likelihood function may be evaluated by applying the Kalman filter. (see. DeJong and Dave, 2007).

In the final step, the Markov Chain Monte Carlo algorithm (MCMC) is applied to draw from the posterior distribution. The algorithm starts from the posterior mode $(\boldsymbol{\theta}^*, \boldsymbol{\phi}^*, \boldsymbol{\omega})$ and returns a series $\{\boldsymbol{\theta}_i, \boldsymbol{\phi}_i, \boldsymbol{\omega}\}_{i=0}^N$. The first part of draws are omitted to ensure that MCMC converges to the "true" posterior distribution. In the empirical application of our algorithm we find 400,000 draws for 2 chains in each model and omit first 300,000 from every chain.

4. Optimal simple rules with uncertainty

In this paper, we propose a new approach to investigate optimal simple rules. In contrast to previous analyses, our approach takes into account the uncertainty about structural parameters. Following an enormous number of previous studies, we focus on *ad hoc* quadratic welfare loss function expressed by[13]:

$$L_t = E_t\left\{\sum_{s=0}^{\infty} \beta^s\, \mathbf{u}'_{\mathbf{t+s}}\mathbf{W}\mathbf{u}_{\mathbf{t+s}}\right\} \tag{31}$$

[13] In our analysis we assume that the central bank is not able to achieve the Pareto efficient equilibrium, since in our model we do not eliminate the monopolistic mark-ups on goods and labor markets to obtain Pareto efficient steady state. Hence, optimal solution of central bank problem should be rather seen as the second best equilibrium.

where: $\mathbf{u_t}$is the vector of central bank target variables and $\mathbf{W} = \operatorname{diag}(\lambda_1, \dots, \lambda_n)$ is diagonal and non-negative weight matrix. Svensson (1999) and Dennis (2004) have shown that in limiting case $\beta \to 1$, the welfare loss function (31) is given by[14]:

$$L_t = \frac{1}{1-\beta} \sum_{i=1}^{n} \lambda_i \operatorname{var}\left(u_t^i\right) \tag{32}$$

where: u_t^i is an i-th component of vector $\mathbf{u_t}$. Our choice of target variables is based on the formal derivation of quadratic welfare loss function proposed by Erceg, Henderson and Levin (2000) and includes inflation rate $\hat{\pi}_t$, output $\hat{y}_t$, real wage $\hat{w}_t$. To make our optimal policy rules more realistic we add the interest rate smoothing term $\Delta\hat{r}_t = \hat{r}_t - \hat{r}_{t-1}$to the welfare loss function.This variable introduces some penalty for large and quick adjustment in interest rate, which should be rather seen as unrealistic. As a result, we assume that

$$\mathbf{u_t} = [\hat{\pi}_t, \hat{y}_t, \hat{w}_t, \Delta\hat{r}_t]' \tag{33}$$

For all target variables, we specify the weights in the objective function. In our benchmark specification, we set $\lambda_\pi = 1$, $\lambda_y = \lambda_w = \lambda_{\Delta r} = 0.5$.

Next, the welfare loss function is used to construct the following optimal control problem of central bank policymaker:

$$\begin{gathered} \min_{\boldsymbol{\phi}} L_t = \min_{\boldsymbol{\phi}} \sum_{i=1}^{n} \lambda_i \operatorname{var}\left(u_t^i\right) \\ \text{st.} \\ \mathbf{A}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega}) E_t\{\mathbf{x_{t+1}}\} + \mathbf{B}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{x_t} + \mathbf{C}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{x_{t-1}} + \mathbf{D}(\boldsymbol{\theta}, \boldsymbol{\phi}, \boldsymbol{\omega})\boldsymbol{\varepsilon_t} = \mathbf{0}, \end{gathered} \tag{34}$$

for fixed weights $\lambda_1, \dots, \lambda_n$ and given functional form of the monetary policy rule (26) which is included into the above constraint. Common approach assumes that vectors of non-policy structural parameters $\boldsymbol{\phi}, \boldsymbol{\omega}$ are known with certainty. Hence the solution of problem (34) is represented by:

$$\boldsymbol{\phi}^{\mathbf{min}} = \phi^{min}(\widehat{\boldsymbol{\theta}}, \boldsymbol{\omega}); L_t^{min} = L_t(\boldsymbol{\phi}^{\mathbf{min}}) = L_t\left(\phi^{min}(\widehat{\boldsymbol{\theta}}, \boldsymbol{\omega})\right) \tag{35}$$

where: $\widehat{\boldsymbol{\theta}}$are known fixed values of $\boldsymbol{\theta}$, $\phi^{min}(\cdot)$ represents functional relationships between structural parameters $\widehat{\boldsymbol{\theta}}, \boldsymbol{\omega}$ and optimal solution $\boldsymbol{\phi}^{\mathbf{min}}$.

[14] It is worth mentioning that we apply precommitment approach in a spirit of (Dennis 2004).

We propose an alternative approach, where the policymaker faces uncertainty regarding model structural parameters. We assume that optimal central bank knows the equations of the model with uncertainty and that the policymaker is aware of the probability distributions of all structural model parameters $\boldsymbol{\theta} \sim p\,(\cdot)$**.** The optimal central bank solves the control problem (34) for all values of random vector $\boldsymbol{\theta}$. Thus in contrast to the standard optimal simple rule problem, we include the uncertainty of structural parameters into the solution $\boldsymbol{\phi}^{\mathbf{min}}$. As a result, our approach allows to find the distributions of optimal monetary policy parameters and minimized welfare loss:

$$\boldsymbol{\phi}^{\mathbf{min}} \sim p\left(\phi^{min}(\boldsymbol{\theta}, \boldsymbol{\omega})\right);\, L_t^{min} \sim p\left(L_t\left(\phi^{min}(\widehat{\boldsymbol{\theta}}, \boldsymbol{\omega})\right)\right) \qquad (36)$$

Although the standard distributions are theoretically interesting, they economic implications may be limited since they are rather not connected with the data. To omit this problem, we propose to use posterior distribution for $\boldsymbol{\theta}$. In contrast two other distributions, posterior distributions measure the uncertainty by taking into account the information from the data and hence they restrict the domain of vector $\boldsymbol{\theta}$ to the empirically relevant values. Incorporating the posterior distribution$\boldsymbol{\theta} \sim p(\cdot\,|\mathbf{Y_T}, \boldsymbol{\omega})$ allows to rewrite the optimal distributions (36) as:

$$\boldsymbol{\phi}^{\mathbf{min}} \sim p\big(\phi^{min}(\boldsymbol{\theta}, \boldsymbol{\omega})|\mathbf{Y_T}; \boldsymbol{\omega}\big);\, L_t^{min} \sim p\big(L_t^{min}(\boldsymbol{\phi}^{\mathbf{min}})|\mathbf{Y_T}; \boldsymbol{\omega}\big) \qquad (37)$$

Computationally, we find the distribution of optimal monetary policy parameters and minimized welfare loss (37) by applying the following steps:

1. Estimate the joint posterior distribution of structural policy and non-policy parameters[15]$p(\boldsymbol{\theta}, \boldsymbol{\phi}, |\mathbf{Y_T}, \boldsymbol{\omega})$.
2. Draw a sequence of vectors $\{(\boldsymbol{\theta}_i, \boldsymbol{\phi}_\mathbf{i})\}_{i=1}^N$ from the jointposterior distribution $p(\boldsymbol{\theta}, \boldsymbol{\phi}|\mathbf{Y}_T, \boldsymbol{\omega})$and take$\{\boldsymbol{\theta}_i\}_{i=1}^N$toobtain a sequence of vectors from $p(\boldsymbol{\theta}|\mathbf{Y}_T, \boldsymbol{\omega})$**.**
3. Solve problem (35) for each vector $\boldsymbol{\theta}_i$, $i = 1,2, \ldots N$ and then use the obtained sequences of solutions $\boldsymbol{\phi}_\mathbf{i}^{\mathbf{min}} = \phi^{min}(\boldsymbol{\theta}_\mathbf{i}, \boldsymbol{\omega})$ and $L_{t,i}^{min} = L_t\left(\phi^{min}(\boldsymbol{\theta}_\mathbf{i},, \boldsymbol{\omega})\right)$, $i = 1,2, \ldots N$, to approximate the distributions of $\boldsymbol{\phi}^{\mathbf{min}}$and L_t^{min}, respectively.

[15] One may expect that in this step policy parameters should be treat as fixed and introduced into the vector $\boldsymbol{\omega}$. However this approach has a important limitation implying that estimates of structural parameters $\boldsymbol{\theta}$ are conditional on policy parameters. We prefer to treat policy parameters as “free” parameters which marginal posterior distributions may be found using data.

In conclusion, our approach allows to obtain distributions of optimal policy rule parameters and minimized welfare loss function. Since we work with distributions of random variables, we compare them by applying stochastic dominance test of order one.

Comparing the distributions of optimal policy reactions and minimized central loss functions

We use stochastic dominance relationships between probability distributions to compare posterior distributions of model structural and monetary policy parameters and distributions of minimized welfare losses.[16] Recall that first-order stochastic dominance (hereafter SD1) of a random variable $\theta_2 \sim F_{\theta_1}$ over $\theta_1 \sim F_{\theta_2}$ (denoted by $\theta_1 \leq_{SD1} \theta_2$) corresponds to $F_{\theta_1}(\theta) \geq F_{\theta_2}(\theta)$, for all $\theta \in R$. Here $F_{\theta_1}, F_{\theta_2}$ are cumulative distribution functions. When $L_1 \leq_{SD1} L_2$ holds for welfare loss distributions associated with two alternative monetary policy rules, then welfare losses summarized by L_2 are at least as large as that in the model with a loss function L_1. In our simulations for a given set of optimal and implementable monetary policy ruleswe focus on determining the best optimal monetary policy rule which generates the smallest distribution of welfare losses with respect to SD1 ordering.

To verify whether one random variable θ_2 statistically dominates over the probability distribution of another variable θ_1, we apply the classical nonparametric Kolmogorov-Smirnov test. We consider the following pair of statistical hypotheses H_0: $\theta_1 =_D \theta_2$ $vs.$ H_1: $\theta_1 \leq_{SD1} \theta_2$ $(\theta_1 \neq_D \theta_2)$. To find statistical arguments for rejecting H_0we calculate the positive part of the Kolmogorov-Smirnov distance

$$D_n^+ = \max_{x \in R}[F_{N,\theta_1}(x) - F_{N,\theta_2}(x)], \tag{38}$$

where F_{N,θ_1}, F_{N,θ_2} are empirical cumulative distribution functions. High values of D_n^+ are in favour of $\theta_1 \leq_{SD1} \theta_2$.

4. Measuring uncertainty of optimal monetary policy response coefficients

In this part of the paper, the algorithm proposed in the previous section is used to measure and analyze the uncertainty of optimal simple monetary policy rules and associated uncertainty of minimized welfare loss. We discuss the implementation of our approach in the case of the Polish economy. First, the description of prior distributions and data used in

[16] Stochastic dominance is very often used for social welfare comparisons (see Deaton (1997)).

estimation are presented. Next, we analyze posteriors. Presented results were obtained by application of the Random Walk Metropolis algorithm (see. Metropolis, et.al., 1953; An, Schorfheide, 2007). Finally, we apply our new approach to answer several interesting questions. We focus on (i) finding the optimal monetary policy rule that minimized the distribution of welfare losses(ii) measuring the influence of adding real output and real wages to policy rules on welfare loss distribution (iii) the importance of interest rate smoothing in optimal monetary policy rules (iv) comparison of estimated and optimized monetary policy rule.

Our simulation result is based on 1'000 randomly draws from the posterior distribution. To ensure comparability of the results, in points (iv) and (v), we limit our attention only to the same 1'000 draws.

Priors and data

We estimate the theoretical model for the Polish economy[17]. This part briefly discusses the prior distributions as well as data used in estimation. As it was presented in previous sections, we divide parameters into 3 vectors: vectors of policy parameters($\boldsymbol{\phi}$) and non-policy estimated parameters ($\boldsymbol{\theta}$) and the vector of calibrated parameters ($\boldsymbol{\omega}$). Table 2 presents our choice of marginal prior distributions for vectors $\boldsymbol{\phi}$ and $\boldsymbol{\theta}$. Our choice seem to be quite standard in comparison with previous works. For all non-policy parameters which belongs to 0-1 interval, we choose beta distributions, whereas for the positive parameters we choose gamma distributions. The exception are standard deviations of measurement errors where we use inverse gamma distributions.

[17] It is worth noting that our model was previously estimated by Rabanal and Rubio-Ramirez for U.S. (2005) and for euro area (2008), as well as Kuchta (2014) and Baranowski and Kuchta (2015) for Polish economy.

Table 2. Prior distributions.

Vector	Parameter			Prior		
	Name	Symbol	Range	Type	Mean	Standard deviation
	Relative risk aversion	δ_c	$(0;\infty)$	Gamma	1.25	0.5
	Price stickiness	θ_p	$(0;1)$	Beta	0.5	0.2
	Wage stickiness	θ_w	$(0;1)$	Beta	0.5	0.2
	Autoregressive parameter – technological shock	ρ_a	$(0;1)$	Beta	0.5	0.2
$\boldsymbol{\theta}$	Autoregressive parameter – preference shock	ρ_b	$(0;1)$	Beta	0.5	0.2
	Standard deviation – technological shock	σ_a	$(0;\infty)$	Gamma	0.1	0.05
	Standard deviation – preference shock	σ_b	$(0;\infty)$	Gamma	0.1	0.05
	Standard deviation – measurement error	σ_{m_r}	$(0;\infty)$	Inverse Gamma	0.01	0.005
	Standard deviation – measurement error	σ_{m_w}	$(0;\infty)$	Inverse Gamma	0.01	0.005
	Monetary policy rule – interest rate smoothing	ρ	$(0;1)$	Beta	0.5	0.2
$\boldsymbol{\phi}$	Monetary policy rule – reaction to inflation	ϕ_π	$(0;\infty)$	Gamma	0.5	0.25
	Monetary policy rule – reaction to output	ϕ_y	$(0;\infty)$	Gamma	0.125	0.05
	Monetary policy rule – reaction to real wage	ϕ_w	$(0;\infty)$	Gamma	0.1	0.05

We use asimilar scheme for policy parameters. We impose beta prior for interest rate smoothing and gamma prior for reaction parameters. Although these priors seem not to be controversial, it is necessary to emphasize some consequences forϕ_π. Chosen prior, together with the Taylor rule (26), implies that we restrict the posterior to a parameter space which is consistent with the Taylor principle. The Taylor principle implies stronger than 1:1 reaction on inflation and it is a condition for the unique solution of linear new Keynesian model under some of considered rules. We decided to impose this requirement to limit the probability of obtaining a huge number of vectors which lies very close to the border of determinacy region. Otherwise, we may obtain biased measurements of uncertainty in parameter $\boldsymbol{\theta}$when we solve problem (35), since we have to eliminate these solutions $\boldsymbol{\phi}^{\mathbf{min}}$ which do not imply the unique equilibrium of the rational expectation system (28)[18].

[18] We do not impose the restriction of Taylor principle during solution of problem (35) in case of all considered model. However we limit our attention only to those solution $\boldsymbol{\phi}^{\mathbf{min}}$which ensure the uniqueness of (28).

The rest of the structural parameters, collected in vector $\boldsymbol{\omega}$,are calibrated. For discount factor (β) we set the value of 0.99. It implies that the annualized real interest rate in the steady state equals 4%. We set the value of 1 for the inverse of Frish labor elasticity (δ_l) and the wage monopolistic mark-up (τ_w) is set at 10%. It implies that labor demand elasticity equals -11.

To obtain the posterior distribution we used quarterly data for Polish economy from 1995:1 to 2015:4. All series come from the Central Statistical Office. We use real GDP per capita as a measure of output, real wage in enterprise sector as a measure of real wages, inflation CPI quarter to quarter as a measure of inflation, and WIBOR 3M (the interbank offer rate) as a measure of nominal interest rate. Before estimation, the real variables (output and real wage) were expressed as logs, seasonally adjusted using the Tramo/Seats procedure and detrended using HP filter. The nominal variables were expressed in percentage and seasonally adjusted (except interest rate). Next, they were divided into two periods, from 1995:1 to 2003:4 we exclude quadratic trend, whereas from 2004:1 to 2015:4 we demean both variables. These transformations are justified by the strong disinflation period in Poland after the transition form centrally planned to market economy, as well as the behavior of the inflation target of Polish National Central Bank. It had substantially decreased from the beginning of the sample up to the end of 2003, and after this period has been constant[19].

Posteriors

Table 3 presents the posteriors estimated for rules: 1 and 8 (see table 1.) in three different dynamic specifications: backward-looking, current-looking and forward-looking, respectively. We report posterior mean and 90% of the highest posterior density (HPD) interval.

Our estimates of structural parameters are consistent among particular models. They indicate rather a high level of price stickiness and low level of wage stickiness. Domination of price stickiness rather than wage stickiness seems to be counterintuitive. However, it is a permanent feature of the DSGE model with constant returns to scale in production and Calvo scheme of price stickiness (see. Smets, Wouters, 2003). Focusing on posterior means the average duration of price contracts range from 4.6 quarters in case of current-looking rules

[19] It is worth noting that the theoretical model assumes that inflation target is constant over time and consistent with zero inflation steady state.

up to 6.3 in the case of forward-looking rules. These estimates are also consistent with some micro-evidence[20].

Table 3. Posterior distributions for rules 1 and 8.

Parameter	Monetary policy rules – backward looking		Monetary policy rules – current looking		Monetary policy rules – forward looking	
	No. 1	No. 8	No. 1	No. 8	No. 1	No. 8
δ_c	2.76 [1.85; 3.67]	2.76 [1.81; 3.62]	1.91 [1.14; 2.70]	1.84 [1.03; 2.64]	2.10 [1.12; 3.00]	2.19 [1.20; 3.12]
θ_p	0.80 [0.75; 0.86]	0.81 [0.75; 0.86]	0.78 [0.71; 0.85]	0.78 [0.71; 0.85]	0.83 [0.79; 0.87]	0.84 [0.80; 0.88]
θ_w	0.61 [0.53; 0.68]	0.61 [0.54; 0.68]	0.58 [0.50; 0.65]	0.58 [0.51; 0.67]	0.60 [0.53; 0.67]	0.61 [0.54; 0.68]
ρ_a	0.54 [0.40; 0.69]	0.54 [0.40; 0.68]	0.56 [0.41; 0.70]	0.56 [0.41; 0.71]	0.48 [0.36; 0.60]	0.45 [0.33; 0.56]
ρ_b	0.75 [0.66; 0.84]	0.77 [0.68; 0.86]	0.78 [0.68; 0.87]	0.79 [0.70; 0.89]	0.72 [0.62; 0.82]	0.71 [0.61; 0.81]
σ_a	0.05 [0.02; 0.08]	0.06 [0.02; 0.09]	0.04 [0.02; 0.07]	0.04 [0.01; 0.07]	0.07 [0.03; 0.11]	0.09 [0.04; 0.14]
σ_b	0.03 [0.02; 0.04]	0.03 [0.02; 0.04]	0.02 [0.01; 0.03]	0.02 [0.01; 0.03]	0.02 [0.01; 0.03]	0.02 [0.01; 0.03]
σ_{m_r}	0.0055 [0.005; 0.006]	0.0058 [0.005; 0.007]	0.0061 [0.005; 0.007]	0.0064 [0.006; 0.007]	0.0051 [0.005; 0.006]	0.0052 [0.004; 0.006]
σ_{m_w}	0.0109 [0.010;0.012]	0.0110 [0.010; 0.012]	0.0108 [0.009; 0.012]	0.0109 [0.010; 0.012]	0.0111 [0.010; 0.013]	0.0113 [0.010; 0.013]
ρ	-	0.09 [0.01; 0.15]	-	0.08 [0.01; 0.15]	-	0.27 [0.06; 0.46]
ϕ_π	0.07 [0.02; 0.12]	0.07 [0.02; 0.12]	0.08 [0.02; 0.13]	0.07 [0.02; 0.13]	0.24 [0.06; 0.39]	0.18 [0.04; 0.30]
ϕ_y	-	0.05 [0.02; 0.08]	-	0.05 [0.02; 0.09]	-	0.05 [0.02; 0.09]
ϕ_w	-	0.07 [0.02; 0.11]	-	0.06 [0.02; 0.11]	-	0.05 [0.01; 0.09]

Notes: posterior mean, 90% HPD in parentheses.

The wage stickiness is substantially lower than price stickiness. Focusing on posterior means the average duration of wage contracts range from 2.4 quarters in case of current-looking rules up to 2.6 quarters in case of backward- and forward-looking rules. Although these estimates may suggest merely noticeable rigidity in the data, the importance of wage stickiness[21] as a part of the DSGE model is strongly supported by empirical results[22].

[20]Macias and Makarski (2013) investigate the average duration of price using microdata for Polish economy. They find that average duration of price contracts equals 11 months and it is higher than in U.S. and smaller than euro area.

[21] It is worth noting that the wage stickiness is extremely important rigidity for optimal monetary policy problems. In the presence of it, the central bank faces trade-off between stabilizing inflation rate, output and wage, whereas in the lack of it, central bank is able to obtain Pareto efficient equilibrium.

Moreover, similar, low wage stickiness was found previously for the Polish economy. For example, Kolasa (2009) estimated the two country DSGE model for Polish and euro area data. The estimates of Calvo-wage stickiness parameter implied that the average duration of wage contracts equals 2.6 quarters, focusing on the posterior mean.

The estimates of monetary policy rule parameters are quite consistent among the backward- and current-looking rules. They indicate a limited reaction to all considered variables as well as the slight effect of interest rate smoothing. Moreover, the introduction of additional variables do not change significantly implied posterior distributions. The forward-looking rules imply a substantially higher level of reaction to inflation and a stronger effect of interest rate smoothing whereas the reactions to wage and output are comparable.

Welfare loss analysis

In this section, we present the results of welfare analysis conducted by means of our algorithm (see Section *Optimal simple rules with uncertainty*). We measure the uncertainty of optimal policy reactions and minimized welfare loss in the estimated New Keynesian model for 24 simple monetary policy rules (see Table 1). Figure 1 shows the highest density intervals (HDI) for all 24 analyzed policy rules grouped by their dynamic specification. The horizontal and vertical axes measure the lower and upper HDI interval, respectively. The closer a point to the origin the smaller welfare losses, whereas the distance from the identity line measures the uncertainty in welfare. We observe that forward-looking rules generatesmaller losses than corresponding backward- and current-looking policy. Forward-looking rules 7 and 8 are the leaders in making the welfare losses to be close to zero level. We compare all pairs of the welfare loss distributions by means of the Kolmogorov-Smirnov test, and we received an unambiguous result that the optimal forward-looking central bank which follows rule 8, given by (40), runs a policy that has the smallest welfare loss distribution.

---

[22] Rabanal and Rubio-Ramirez (2005) compared different specifications of closed economy, small-scale DSGE model using data for U.S. They found that Erceg, Henderson and Levine type model is preferred by the data over the models omitting the wage stickiness, even if price indexation is introduced. This observation was confirmed by them in case of euro area (Rabanal, Rubio-Ramirez, 2008) and Kuchta (2014) in case of Poland. Moreover, Smets and Wouters (2007) investigated the empirical importance of nominal and real rigidities in medium-scale DSGE model in a spirit of Christiano, Eichenbaum and Evans (2005). Their results also suggest importance of wage stickiness. Finally, similar results were found by Adolfson et.al. (2007) in a open economy DSGE model.

$$\hat{r}_t = \underset{[0.85;\,3.11]}{2.00}\ \hat{r}_{t-1} + \underset{[1.04;\,1.70]}{1.41}\ E_t\{\hat{\pi}_{t+1}\}$$
$$+ \underset{[1.69;\,6.57]}{4.18}\ E_t\{\hat{y}_{t+1}\} + \underset{[1.31;\,4.27]}{2.74}\ E_t\{\widehat{w}_{t+1}\} \tag{39}$$

The best optimal forward-looking monetary policy rule, (39), admits the following features. First, it is a super-inertial interest rate rule (see: (Giannoni and Woodford 2002), Schmitt-Grohe, Uribe 2004). Thus the optimal monetary policy instrument is a function of lagged values of the policy instrument, and moreover the persistence parameter ρ_r takes values higher than 1. Second,we receive moderate reaction to inflation, which is consistent with the results of Taylor 1993, and we report that the best optimal monetary policy rule admits very aggressive reaction to both output gap and real wages.

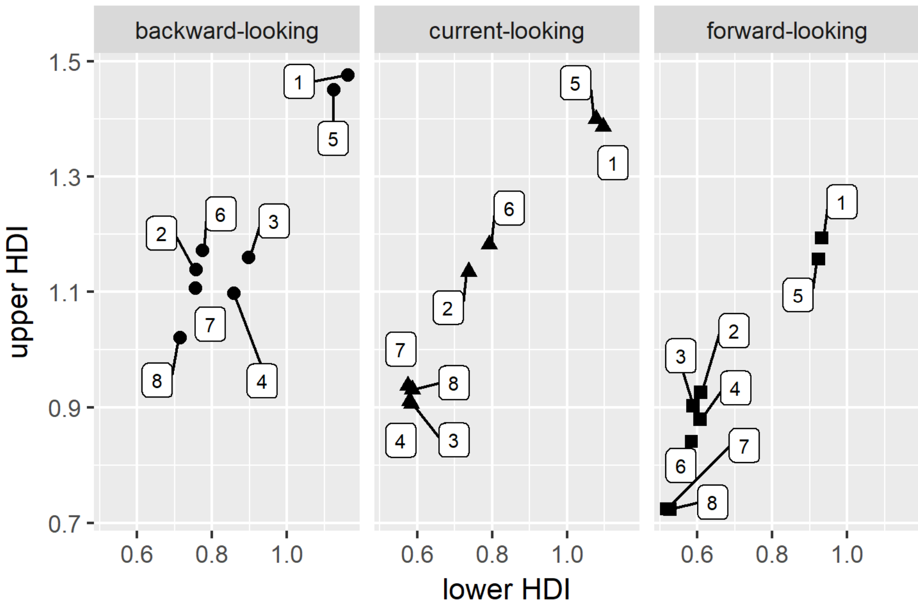


Figure 1. 90% HDI for distributions of minimized welfare losses form eight optimal Taylor-type rules (see Table 1) with three dynamic monetary policy specification: backward-, current- and forward-looking.

The influence of real variables on welfare loss and optimal monetary policy rules

This part evaluates the consequences of using different measures of economic fluctuations in optimal monetary policy rules. Our analysis starts with a comparison of the strict inflation targeting policy, where the interest rate responses only on inflation, against the flexible inflation targeting. The latter strategy of the central bank implies that the interest rate is set as a function of the inflation rate and also other variables, including measures of real activity[23]. In our work, we consider two of them: commonly used output and real wage which may be seen as alternative and more connected with labor market measure of business cycle fluctuations. In the comparison, we apply the nonparametric Kołmogorov-Smirnov test (KS test) to the optimal distributions of welfare losses (38) generated under alternative monetary policy rule (see table 1.). Results of the comparison are presented in Table 4.

Intuitively, the strict inflation targeting is not able to reduce welfare loss to the levels comparable with an alternative monetary policy rule. Results of KS test indicate that welfare losses implied by rule no. 1 and no. 5 are statistically higher than losses obtained from rules no. 2 – 4 and no. 6 – 8. These results hold for backward-, forward- and current-looking rules. Focusing on means of welfare losses, incorporating real variables into optimal monetary policy rules allows reducing welfare loss by depending on the rule: 22-32% in case of backward-looking rules, 21-41% in case of current-looking rules and 27-39% in case of forward-looking rules.

Next, we investigate the influence of the real variables on the distribution of welfare losses. Similarly to previous cases, we apply the KS test. The results are given in table 5. Overall, they are mixed. For backward-looking rules, incorporating real wages into optimal monetary policy rule (no. 2) causes statistically lower welfare loss than rule including output (no. 3), even if real wages are also reaction variable (rule no. 4). However, the changes in welfare loss are small (2-6% depending on the rule). These results do not hold in rules with interest rate smoothing. Incorporating this mechanism into optimal monetary policy rules causes that rule with output (no. 7) generates statistically lower welfare loss than rule reacting to real wage (rule no. 6). The rule including both variables generates statistically the lowest welfare loss. Moreover, similarly to rules without interest rate smoothing, the changes in welfare losses are rather small (4-9% depending on rule).

[23] It is worth noting that our interpretation of strict and flexible inflation targeting is different from proposed by (Svensson 1997), who defines strict and flexible inflation targeting in terms of welfare loss function (33) instead of monetary policy rule (26).

Table 4. The influence of real variables on the distribution of welfare losses. Results of Kolmogorov-Smirnov test (KS test) for distributions of welfare loss.

Type of rule	Model 1					Model 2					KS test* (p-value)
	No. of rule	Welfare loss				No. of rule	Welfare loss				
		mean	median	5%	95%		mean	median	5%	95%	
backward-looking ($i = -1$)	1	1.31	1.30	1.16	1.48	2	0.96	0.95	0.76	1.14	0.000
	1	1.31	1.30	1.16	1.48	3	1.02	1.01	0.90	1.16	0.000
	1	1.31	1.30	1.16	1.48	4	0.98	0.98	0.86	1.10	0.000
	5	1.29	1.28	1.12	1.45	6	0.97	0.98	0.77	1.17	0.000
	5	1.29	1.28	1.12	1.45	7	0.93	0.92	0.75	1.11	0.000
	5	1.29	1.28	1.12	1.45	8	0.88	0.87	0.72	1.02	0.000
current-looking ($i = 0$)	1	1.25	1.24	1.10	1.39	2	0.95	0.94	0.74	1.14	0.000
	1	1.25	1.24	1.10	1.39	3	0.74	0.72	0.58	0.91	0.000
	1	1.25	1.24	1.10	1.39	4	0.75	0.74	0.58	0.91	0.000
	5	1.24	1.22	1.08	1.40	6	0.98	0.98	0.79	1.18	0.000
	5	1.24	1.22	1.08	1.40	7	0.77	0.75	0.58	0.94	0.000
	5	1.24	1.22	1.08	1.40	8	0.76	0.75	0.59	0.93	0.000
forward-looking ($i = 1$)	1	1.07	1.07	0.93	1.19	2	0.78	0.77	0.61	0.93	0.000
	1	1.07	1.07	0.93	1.19	3	0.76	0.75	0.59	0.90	0.000
	1	1.07	1.07	0.93	1.19	4	0.74	0.73	0.61	0.88	0.000
	5	1.03	1.03	0.92	1.16	6	0.72	0.71	0.58	0.84	0.000
	5	1.03	1.03	0.92	1.16	7	0.64	0.63	0.52	0.72	0.000
	5	1.03	1.03	0.92	1.16	8	0.63	0.62	0.53	0.72	0.000

Notes: *the KS test is conducted for hypotheses $H_0: L_1 =_D L_2$ vs. $H_1: L_2 \leq_{SD1} L_1$, where L_1 and L_2 are minimized welfare loss in Model 1 and Model 2. Each row presents also some statistics about distributions of L_1 and L_2. The shaded cells shows model which generates statistically smaller welfare loss distribution.

Different pattern exhibits if we will analyze the optimal current-looking monetary policy rule. Incorporating output into policy rule without interest rate smoothing (no. 3) causes the statistically lower level of welfare loss (by 22%) than rule which includes real wages (no 2.). Moreover, taking into consideration both variables (rule no. 4) causes statistically higher welfare loss than considering only output (rule no. 3), although the change in welfare loss is very small. These results hold also for rules with interest rate smoothing. Similarly, rule with output indicates statistically lower welfare losses than rule with real wage (by 21%), whereas the difference in welfare losses caused by the broadest rule (no. 8) and rule only with output is statistically insignificant.

Table 5. The influence of real variables on distribution of welfare losses. Results of Kolmogorov-Smirnov test (KS test) for distributions of welfare loss.

Type of rule	Model 1					Model 2					KS test* (p-value)
	No. of rule	Welfare loss				No. of rule	Welfare loss				
		mean	median	5%	95%		mean	median	5%	95%	
backward-looking $(i = -1)$	2	0.96	0.95	0.76	1.14	3	1.02	1.01	0.90	1.16	0.000
	2	0.96	0.95	0.76	1.14	4	0.98	0.98	0.86	1.10	0.000
	3	1.02	1.01	0.90	1.16	4	0.98	0.98	0.86	1.10	0.000
	6	0.97	0.98	0.77	1.17	7	0.93	0.92	0.75	1.11	0.000
	6	0.97	0.98	0.77	1.17	8	0.88	0.87	0.72	1.02	0.000
	7	0.93	0.92	0.75	1.11	8	0.88	0.87	0.72	1.02	0.000
current-looking $(i = 0)$	2	0.95	0.94	0.74	1.14	3	0.74	0.72	0.58	0.91	0.000
	2	0.95	0.94	0.74	1.14	4	0.75	0.74	0.58	0.91	0.000
	3	0.74	0.72	0.58	0.91	4	0.75	0.74	0.58	0.91	0.006
	6	0.98	0.98	0.79	1.18	7	0.77	0.75	0.58	0.94	0.000
	6	0.98	0.98	0.79	1.18	8	0.76	0.75	0.59	0.93	0.000
	7	0.77	0.75	0.58	0.94	8	0.76	0.75	0.59	0.93	0.100
forward-looking $(i = 1)$	2	0.78	0.77	0.61	0.93	3	0.76	0.75	0.59	0.90	0.000
	2	0.78	0.77	0.61	0.93	4	0.74	0.73	0.61	0.88	0.000
	3	0.76	0.75	0.59	0.90	4	0.74	0.73	0.61	0.88	0.000
	6	0.72	0.71	0.58	0.84	7	0.64	0.63	0.52	0.72	0.000
	6	0.72	0.71	0.58	0.84	8	0.63	0.62	0.53	0.72	0.000
	7	0.64	0.63	0.52	0.72	8	0.63	0.62	0.53	0.72	0.001

Notes: *the KS test is conducted for hypotheses $H_0: L_1 =_D L_2$ vs. $H_1: L_2 \leq_{SD1} L_1$, or $H_1: L_2 \geq_{SD1} L_1$ where L_1 and L_2 are minimized welfare loss in Model 1 and Model 2. Each row presents also some statistics about distributions of L_1 and L_2. The shaded cells shows model which generates statistically smaller welfare loss distribution.

The optimal forward-looking monetary policy rules present intuitively more consistent pattern. Focusing on rules without interest rate smoothing, the rule with output (no. 3) allows for statistically lower welfare loss than rule with real wage (no. 2), but including both variables (rule no. 4) results in the lowest welfare loss, although changes in these losses are small (3-5% depending on rule). Similar results hold for rules with interest rate smoothing. For them also incorporating output (rule no. 7) instead of wage (rule no 6.) significantly lowers the welfare loss and the rule no. 8 allows to obtain the smallest welfare losses. Moreover, in this group of rules changes in welfare losses are higher and range from 2% to 13% depending on the rule.

In conclusions, our analysis shows that the strict inflation targeting seems to be rather a non-optimal strategy for policymakers in comparison with flexible inflation targeting, if measures of uncertainty are taken into consideration. This result seems to be quite intuitive since the central bank is rather interesting in observing and reacting to broad set of macroeconomic indicators. Moreover, the choice of real variables, which should be included in monetary policy rule is not very obvious. Surprisingly, the more sophisticated rules not necessarily should ensure the minimal welfare loss. The proper choice of optimal simple monetary policy rule seems to depend on the dynamic specification[24] of it as well as including the interest rate smoothing mechanism. Moreover, the consequences of applying improper rule are also differentiated. For some pairs of rules changes in welfare losses are barely worth mentioning, whereas for other pairs they are higher than 20%.

The importance of interest rate smoothing in optimal monetary policy rules

This part analyzes the consequences of introducing interest rate smoothing into optimal monetary policy rule. Interest rate smoothing may be seen as some sign of cautiousness in monetary policy since it allows for thegradual response of nominal interest rate. In the comparison of different rules we apply once more the KS test. Table 6. presents the results.

Overall our results are mixed. Focusing on backward-looking rules, the interest rate smoothing significantly lowers welfare loss in case of strict inflation targeting rule (no 1. vs. no. 5), although the change in welfare loss is slight. For flexible inflation targeting rules, the introduction of interest rate smoothing lowers significantly welfare losses only in case of rules including reaction to output and changes in welfare loss are noticeable (9-10% depending on rule). For rules considering only real wages (no. 2 vs. no. 6), the incorporating of interest rate smoothing significantly increases welfare loss, but by very small value.

Focusing on current-looking rules, the positive influence of interest rate smoothing on welfare loss is doubtful. Although interest rate smoothing statistically decreases the welfare loss for strict inflation targeting rule, it statistically increases this loss for flexible inflation targeting rules. It is true for rules including output, real wage as well as both

[24] Intuitively, particular dynamic specifications of theTaylor rule have advantages and disadvantages. For example, the forward-looking rules may be seen as the central bank willing to react faster on some shocks, since they contain future values of reaction variables. On the other hand, implementing of such rule is not obvious and may be connected with additional errors, since central bank needs to forecast future values of particular variables. Similarly, backward-looking rules are easy to implement, but reaction of central bank may be not adequate to current circumstances, since they take into consideration only previous values of variables.

variables. However, in all considered cases, changes in welfare losses are small. They range from 1% to 4% depending on rule.

Table 6. The influence of interest rate smoothing on the distribution of welfare loss. Results of Kolmogorov-Smirnov test (KS test) for the distribution of welfare losses.

Type of rule	Model 1					Model 2					KS test* (p-value)
	No. of rule	Welfare loss				No. of rule	Welfare loss				
		mean	median	5%	95%		mean	median	5%	95%	
backward-looking ($i = -1$)	1	1.31	1.30	1.16	1.48	5	1.29	1.28	1.12	1.45	0.000
	2	0.96	0.95	0.76	1.14	6	0.97	0.98	0.77	1.17	0.000
	3	1.02	1.01	0.90	1.16	7	0.93	0.92	0.75	1.11	0.000
	4	0.98	0.98	0.86	1.10	8	0.88	0.87	0.72	1.02	0.000
current-looking ($i = 0$)	1	1.25	1.24	1.10	1.39	5	1.24	1.22	1.08	1.40	0.000
	2	0.95	0.94	0.74	1.14	6	0.98	0.98	0.79	1.18	0.000
	3	0.74	0.72	0.58	0.91	7	0.77	0.75	0.58	0.94	0.000
	4	0.75	0.74	0.58	0.91	8	0.76	0.75	0.59	0.93	0.013
forward-looking ($i = 1$)	1	1.07	1.07	0.93	1.19	5	1.03	1.03	0.92	1.16	0.000
	2	0.78	0.77	0.61	0.93	6	0.72	0.71	0.58	0.84	0.000
	3	0.76	0.75	0.59	0.90	7	0.64	0.63	0.52	0.72	0.000
	4	0.74	0.73	0.61	0.88	8	0.63	0.62	0.53	0.72	0.000

Notes: *the KS test is conducted for hypotheses $H_0: L_1 =_D L_2$ vs. $H_1: L_2 \leq_{SD1} L_1$, or $H_1: L_2 \geq_{SD1} L_1$ where L_1 and L_2 are minimized welfare loss in Model 1 and Model 2. Each row presents also some statistics about distributions of L_1 and L_2. The shaded cells shows model which generates statistically smaller welfare loss distribution.

These results seem to be quite counterintuitive since in our welfare loss function we include a penalty for too often and too sharp changes in interest rate. Finally, for forward-looking rules incorporating of interest rate smoothing statistically decreases the welfare loss in all considered rules. Moreover, changes in welfare losses are differentiated. Focusing on the expected value of optimal distributions, they range from 4% in case of strict inflation targeting rule up to 16% in case of rule which includes inflation and output (no. 3 vs. no. 7).

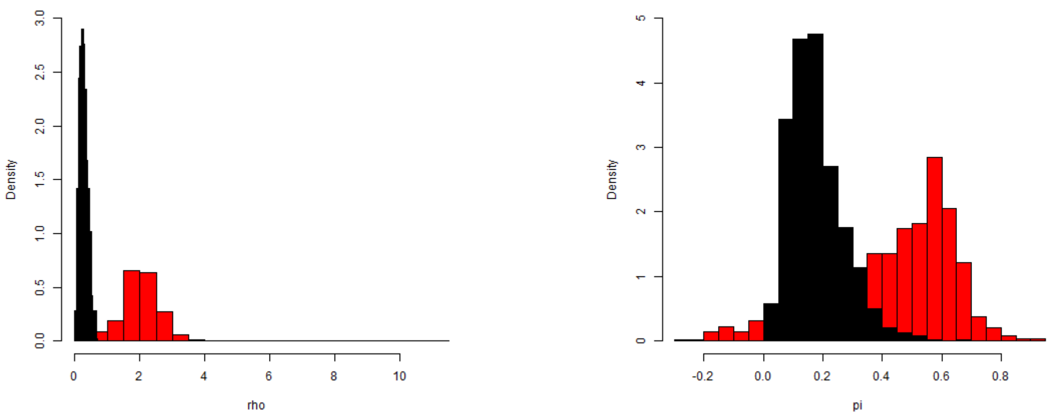
In conclusion, adding the interest rate smoothing term into monetary policy rule is not necessarily consistent with optimal behavior. Although caution in policy decision making is naturally desired feature of the responsible central bank, especially in the uncertain environment, our results of simulation not necessary support this opinion. Contrary, in some cases, results of KS test rather show that gradual response of interest rate may not be an optimal behavior, if the central bank reacts to some real variables. However, the changes of welfare loss are rather small in this cases, up to 4% of welfare loss. As a result, perceiving the interest rate smoothing as a permanent feature of optimal central bank behavior in uncertain environment may be not necessary correct.

Comparison of estimated and optimized monetary policy rules

In this section, we compare the distributions of parameters describing the central bank's reactions yielding from the best optimal monetary policy rule and corresponding empirical forward-looking rule no. 8 (see Figure 2). Furthermore, we also perform a statistical comparison of the minimized and empirical welfare loss distributions. Finally, we check whether there is a significant difference between the probability distributions of inflation variances derived from the empirical model and the model with the best optimal rule.

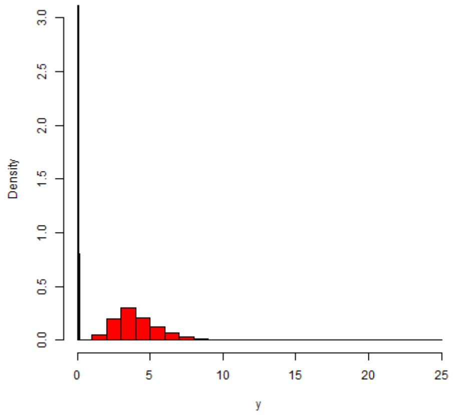
The Kolmogorov-Smirnov test confirms that the best optimal rule generates significantly lower welfare loss, for this rule the median of welfare losses is twice lower than the median resulting from the application of the empirical rule (see Figure 2 e)). The receive this welfare loss reduction the optimal central bank should increase all reactions on both lagged interest rates and on all expected deviation of inflation, output and wages form their steady-state values (see Figure 2 a)-d)).

One of the fundamental question concerning the implementation of the optimal monetary policy rules is whether they are able to sufficiently reduce the variance of inflation which is important for central banks following strict inflation targeting strategy. It turns out that the distributions of inflation variances resulting from the application of optimal and empirical rules do not differ statistically(see Figure 2 f)).This means that the best optimal rule is able to control at the same level as the empirical rule the fluctuations of inflation.

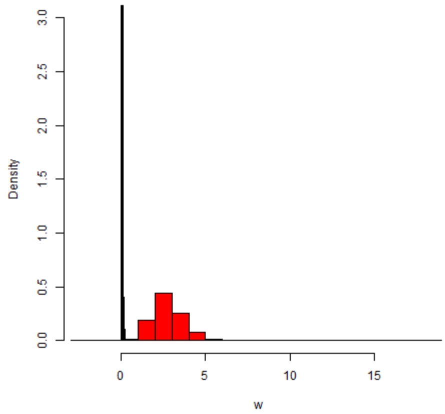


a) $\rho_r \leq_{SD1} \rho_r^{min}$,KS test*, p-value < 0.001

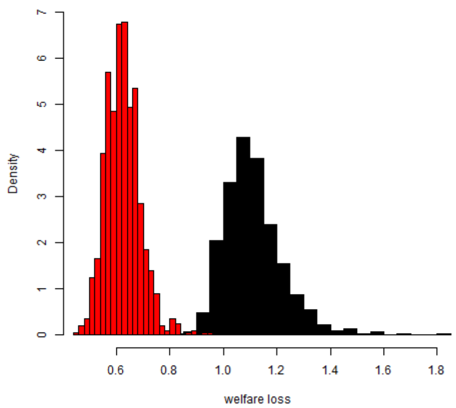
b) $\phi_\pi \leq_{SD1} \phi_\pi^{min}$, KS test*, p-value < 0.001



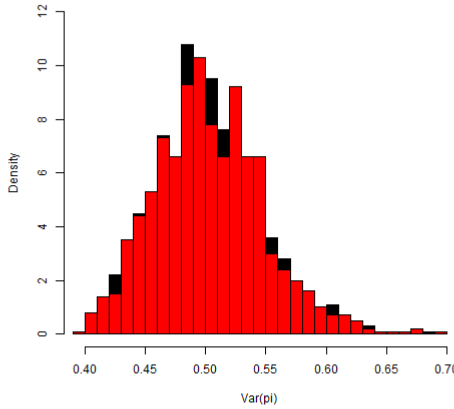
c) $\phi_y \leq_{SD1} \phi_y^{min}$, KS test*, p-value < 0.001*



d)$\phi_w \leq_{SD1} \phi_w^{min}$,KS test*, p-value < 0.001



e)$L^{min} \leq_{SD1} L$, KS test*, p-value < 0.001



f) $Var(\pi_t^{min}) =_D Var(\pi_t)$, KS test*, p-value > 0.5

Figure 2. Probability density functions (pdfs) of rule parameters $(\rho_r, \phi_\pi, \phi_y, \phi_w)$, $(\rho_r^{min}, \phi_\pi^{min}, \phi_y^{min}, \phi_w^{min})$, pdfs of welfare loss distributions L, L^{min} ,pdfs of inflation variances$Var(\pi_t), Var(\pi_t^{min})$ for the estimated and the best optimal monetary policy rules, respectively.

Notes: *the KS test is conducted for hypotheses $H_0: X_1 =_D X_2$ vs. $H_1: X_2 \leq_{SD1} X_1$, or $H_1: X_2 \geq_{SD1} X_1$ where X_1 and X_2 are corresponding random variables.
the empirical forward-looking monetary policy rule:
$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1+\phi_\pi)E_t\hat{\pi}_{t+1} + \phi_y E_t\hat{y}_{t+1} + \phi_w E_t\widehat{w}_{t+1}$– black shaded area,
the best optimal forward-looking monetary policy rule:
$\hat{r}_t = \rho_r^{min}\hat{r}_{t-1} + (1+\phi_\pi^{min})E_t\hat{\pi}_{t+1} + \phi_y^{min}E_t\hat{y}_{t+1} + \phi_w^{min}E_t\widehat{w}_{t+1}$– red shaded area.

Conclusions

This paper proposes a new algorithm of solving optimal precommitment simple policy rule with uncertainty. In contrast to previous works, our algorithm enables us to derive the distributions of optimal reaction parameters and implied welfare loss for a given functional form of the monetary policy rule. Minimization of welfare loss function is performed under the assumption that structural parameters of the underlying model are random variables. The distributions of these variables measure uncertainty and come from the Bayesian inference performed on the data. Since our approach treats optimization results as random variables we apply the first order stochastic dominance ordering to compare particular results and draw some useful conclusions.

In the application, we use an estimated version of the sticky price and wages model to ask several questions. Firstly, we perform the welfare analysis and find the optimal simple rule for the Polish economy. Quite intuitively, it is a forward-looking rule with interest rate smoothing and reaction to inflation, output and real wage – these variables were included into welfare loss function. Secondly, we compare two different strategies of conducting monetary policy: strict inflation targeting in which the policymaker sets interest rate as a function of inflation and flexible inflation targeting. We show that the strict inflation targeting seems to be rather a non-optimal strategy for policy makers who take into consideration the measurement of parameter uncertainty. Thirdly, considering different combinations of two alternative sets of reactions variables, we show that the full monetary policy rules, which react on all variables, not necessarily ensure the minimal welfare loss even in the simple DSGE model like ours. Next, we perform a similar analysis for interest rate smoothing term and show that gradual response of interest rate is not necessarily optimal in our environment. Finally, we compare the estimated and optimized interest rate rules and find significant differences between posteriors obtained from data and optimal distributions. Moreover, we show that although optimal interest rate rule exhibits a much stronger reaction to real variables than reaction found in the data, it is not necessarily contradictory to the main aim of monetary policy – stabilization of inflation rate.

Our approach seems to be quite promising and flexible since (i) it distinguishes between parameters treated as known numbers and parameter uncertainty, (ii) it allows to asses different source of uncertainty and (iii) it fits to a broad range of macroeconomic problems which may be rewritten as LQ optimization problems.

References:

1. Adolfson M., Laseen S., Linde J., Villani M. (2007), Bayesian estimation of an open economy DSGE model with incomplete pass-through, *Journal of International Economics*, No. 72, pp. 481 – 511.
2. Altug S. (1989), Time-to-Build and Aggregate Fluctuations: Some New Evidence, *International Economic Review*, Vol. 30, No. 4, pp. 889 – 920.
3. An S., Shorfheide F. (2007), Bayesian Analysis of DSGE Models, *Econometric Review*, Vol. 26, No. 2 – 4, pp. 113 – 172.
4. Baranowski P., Kuchta Z. (2015), Changes in nominal rigidities in Poland – a regime switching DSGE perspective, *Lodz Economics Working Papers*, No. 6/2015.
5. Baranowski P., Górajski M., Malaczewski M., Szafrański G. (2016), Inflation in Poland under State-Dependent Pricing, *Ekonomicky casopis/Journal of Economics*, Vol. 64, No. 10, pp. 923 – 936.
6. Blinder, A. S. (1999). *Central Banking in Theory and Practice*. Mit press.
7. Calvo G.A. (1983), Staggered Prices in a Utility-Maximizing Framework, *Journal of Monetary Economics*, No. 12, pp. 383 – 398.
8. Chow, G. C.(1975),*Analysis and Control of Dynamic Economic Systems*. Wiley.
9. Christiano L.J., Eichenbaum M. (1992), Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations, *The American Economic Review*, Vol. 82, No. 3, pp. 430 – 450.
10. Christiano L.J., Eichenbaum M., Evans Ch.L. (2005), Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy*, Vol. 113, No. 1, pp. 1 – 45.
11. Clarida, Richard, Jordi Gali, and Mark Gertler. 1999. The Science of Monetary Policy: A New Keynesian Perspective.
12. Cogley, T., De Paoli, B., Matthes, C., Nikolov, K., and Yates, T. (2011) A Bayesian approach to optimal monetary policy with parameter and model uncertainty, *Journal of Economic Dynamics and Control*, 35(12):2186–2212.
13. DeJong D.N., Dave Ch. (2007), Structural Macroeconometrics, Princeton University Press, Princeton
14. Deaton, A. (1997): The Analysis of Household Surveys: A Microeconometric Approach to Development Policy. Baltimore: John Hopkins University Press.
15. Dennis R. (2004) Solving for Optimal Simple Rules in Rational Expectations Models,Journal of Economic Dynamics and Control 28(8), pp.1635–60.
16. Dixit A.K., Stiglitz J.E. (1977), Monopolistic Competition and Optimum Product Diversity, The American Economic Review, No. 67, No. 3, pp. 297 – 308.
17. Duffie D., Singleton K.J. (1993), Simulated Moments Estimation of Markov Models of Asset Prices, Econometrica, Vol. 61, No. 4, pp. 929 – 952.
18. Erceg Ch. J., Henderson D.W., Levin A.T. (2000), Optimal Monetary Policy with Staggered Wage and Price Contracts, Journal of Monetary Economics, No. 46, pp. 281 – 313.

19. Fernandez-Villaverde J. (2010), The Econometrics of DSGE Models, SERIEs Journal of Spanish Economic Association, No. 1, pp. 3 – 49.
20. Giannoni, M. P. (2002), Does Model Uncertainty Justify Caution? Robust Optimal Monetary Policy in a Forward-Looking Model, Macroeconomic Dynamics 6(1)
21. Giannoni, M. P. (2007), Robust Optimal Monetary Policy in a Forward-Looking Model with Parameter and Shock Uncertainty,Journal of Applied Econometrics 22(1):179–213.
22. Giannoni M. P. and Woodford M. (2002) Optimal interest rules: 1. General theory, NBER, no.9419.
23. Goodhart, C. A. (1999), Central Bankers and Uncertainty, in Proceedings-British Academy, vol. 101, pp. 229–72
24. Górajski M. (2017) Robust Monetary Policy in a Model of the Polish Economy: Is the Uncertainty Responsible for the Interest Rate Smoothing Effect?,Computational Economics 1–28.
25. Greenspan, Alan. (2004), Risk and Uncertainty in Monetary Policy,American Economic Review, vol. 94(2), pp. 33–40.
26. Guerron-Quintana P.A., Nason J.M. (2012), Bayesian Estimation of DSGE Models, Federal Reserve Bank of Philadelphia Working Papers, No. 12-4.
27. Justiniano A., Preston B. (2010), Monetary policy and uncertainty in an empirical small open-economy model, J. Appl. Econ. 25, pp. 93–128
28. Kendrick, David A. (2005), Stochastic Control for Economic Models: Past, Present and the Paths Ahead, Journal of Economic Dynamics and Control 29(1–2), pp. 3–30.
29. Kolasa M. (2009), Structural Heterogeneity or Asymmetric Shocks? Poland and the Euro Area through the Lens of a Two-Country DSGE Model, Economic Modelling, No. 29, pp. 1245 – 1269.
30. Krajewski P. (2015), Effectiveness of the Fiscal Policy in Stimulating Economy: The Case of Poland, Transformation in Business & Economics, Vol. 14, No. 2, pp. 53 – 67.
31. Kuchta Z. (2014), Nominal wage rigidities in small-scale DSGE models: An empirical Analysis for the Polish economy, National Economy, No. 6, pp. 31 – 56. (text in Polish)
32. Levine, P., McAdam, P., and Pearlman, J. (2012). Probability Models and Robust Policy Rules, European Economic Review, 56(2), 246 – 262.
33. Macias P., Makarski K. (2013), Stylizowane fakty of cenachkonsumenta w Polsce (Stylized facts about consumer prices in Poland), Materiały i Studia, Nr 295, National Bank of Poland (text in polish)
34. Metropolis N., Rosenbluth A.W., Rosenbluth M.N., Teller A.H., Teller E. (1953), Equation of State Calcuations by Fast Computing Machines, The Journal of Chemical Physics, Vol. 21, No. 6, pp. 1087 – 1092.
35. Onatski A. and Stock J. H. (2002) Robust Monetary Policy Under Model Uncertainty in a Small Model of the U.S. Economy,Macroeconomic Dynamics, 6(1), pp. 85–110.
36. Onatski, A. and Williams N. (2003) Modeling Model Uncertainty,Journal of the European Economic Association, 1(5), pp.1087–1122.
37. Poole W. (1998) A Policymaker Confronts Uncertainty,Federal Reserve Bank of St. Louis Review 80.

38. Rabanal P., Rubio-Ramirez J.F. (2005), Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach, Journal of Monetary Economics, No. 52, pp. 1151 – 1166.

39. Rabanal P., Rubio-Ramirez J.F. (2008), Comparing New Keynesian Models in the Euro Area: A Bayesian Approach, Spanish Economic Review, No. 10, pp 23 – 40.

40. Schmitt-Grohe S., Uribe M. (2004a), Optimal Operational Monetary Policy in the Christiano-Eichenbaum-Evans Model of the U.S. Business Cycle, NBER Working Paper, No. 10724.

41. Schmitt-Grohe S., Uribe M. (2004b), Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Functions, Journal of Economic Dynamics & Control, No. 28, pp. 755 – 775.

42. Schmitt-Grohe S., Uribe M. (2005), Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model. Expanded Version, NBER Working Papers Series, No. 11417.

43. Smets F., Wouters R. (2003), An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area, Journal of European Economic Association, Vol. 1, No. 3, pp. 1123 – 1175.

44. Smets F., Wouters R. (2007), Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach, The American Economic Review, Vol. 97, No. 3, pp. 586 – 606.

45. Smith A.A. (1993), Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregression, Journal of Applied Econometrics, Vol. 8, No. S1, pp. S63 – S84.

46. Svensson L.E.O. (1997) Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets,European Economic Review 41(6), pp. 1111–1146.

47. Svensson, L.E.O. (1999) Inflation Targeting as a Monetary Policy Rule,Journal of Monetary Economics, 43(3), pp. 607–654.

48. Taylor J.B. (1993), Discretion Versus Policy Rules in Practice, Carnegie-Rochester Conference Series on Public Policy, No. 39, pp. 195 – 214.

49. Taylor, J.B. (1999), Monetary policy rules. University of Chicago Press.

50. Taylor J.B. and Williams J.C. (2010), Simple and Robust Rules for Monetary Policy, Chapter 15 in Handbook of Monetary Economics, vol. 3, , Elsevier

51. Tinbergen J. (1952), On the Theory of Economic Policy, Amsterdam: North Holland Pub. Co.

52. Woodford M. (2003), Interest and Prices. Princeton University.

53. Yun T. (1996), Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles, Journal of Monetary Econo*mics*, No. 37, pp. 345 – 370.