



# Scaling violation and the inelasticity of very high energy proton–proton interactions

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## ABSTRACT

The pseudorapidity measurements at LHC, although in the central region only, allows us to perform preliminary tests of the multiparticle production extrapolation formula inspired by the analysis of recent cosmic ray data. Feynman scaling violation in the form proposed originally by Wdowczyk and Wolfendale in the 1970s has been applied to the Pierre Auger Observatory and the Hi-Res group measurements. The consistency of the average extensive air shower development with the hypothesis of protons being the primary particles, as indicated by anisotropy observations, was found for a smooth rise of the scaling violation parameter. We have shown that the longitudinal momenta of the produced particles determined inclusively as rapidity (pseudorapidity) distributions measured by LHC experiments follow the same universal high energy distribution scaled respectively. The high degree of Feynman scaling violation is confirmed. The decrease of the very high energy interaction inelasticity suggested by the cosmic ray data analysis is found to be consistent with LHC measurements up to 7 TeV.

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## 1. Introduction

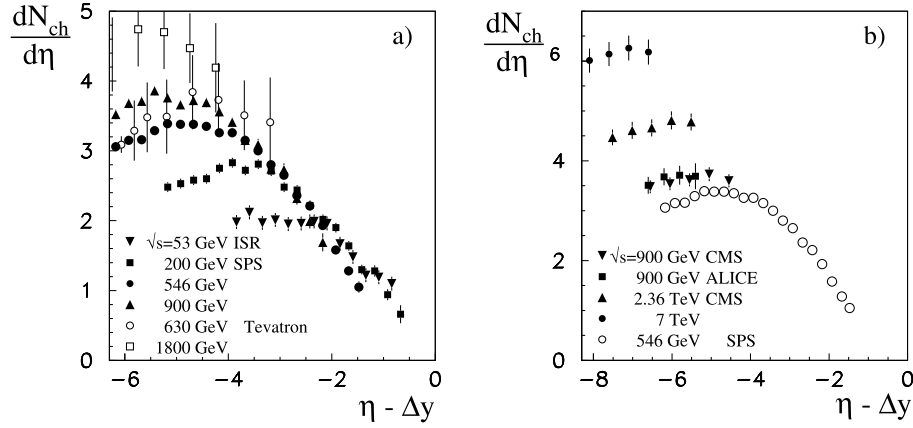
The inclusive description of minimum bias LHC events is not as spectacular as, for example, Higgs hunting, but it is essential for other very important scientific endeavours. One of them is the Ultra High-Energy Cosmic Ray (UHECR) problem and the answer to the question of the existence of the Greisen–Zatsepin–Kuzmin (GZK) cut-off [1]. The origin and nature of cosmic rays has been studied for almost exactly 100 years. Great experimental effort has been made recently by two groups: the Pierre Auger Observatory [2] and the Hi-Res experiment [3]. There has been progress, but the answers are still not decisive. The cosmic rays of energies of about  $10^{20}$  eV, if they are protons, should not reach us from cosmological distances. Our knowledge about the nature of UHECR is based on the observation of giant Extensive Air Showers (EAS) – cascades of secondary particles created in the atmosphere when a single atomic nucleus (proton in the simplest case) enters from above. It is expected that the EAS initiated by protons and iron nuclei should differ. This difference is determined by the rate of energy dissipation. Thus it depends strongly on the distribution of secondaries produced in the forward direction and on the nature of primary particle: its atomic mass. The long-lasting discussions on the primary cosmic ray mass composition at the very end of the cosmic ray energy spectrum, in the so-called “ankle” region ( $E_{\text{lab}} > 10^{18}$  eV), cannot be conclusive also because of the lack

of the more exact knowledge of the very high energy interaction physics, this makes the importance of the high energy proton fragmentation even greater for cosmic ray physicists, astronomers and cosmologists.

Searching for regularities and a phenomenological description of the multiparticle production model is as old as the modeling in high-energy physics itself. Starting from the simple Fermi thermodynamical model, to the first parton (quark) model by Feynman, the model extrapolation to much higher, cosmic ray, energies was one of the most important and most wanted model predictions. It is usually in the form of a kind of scaling. The idea of limited fragmentation [4] applied to the quark-jet hadronization led to introduction of the Feynman scaling variable  $x_F$  and the universal fragmentation function  $f(x_F, s) = f_F(x_F)$  [5]. This brilliant idea worked well for the first collider experiments up to  $\sqrt{s} \sim 60$  GeV. However, when applied to cosmic ray EAS development, it was questioned already at the “knee” energies of  $E_{\text{lab}} \sim 10^{15}$  eV. The SPS ( $\sqrt{s} \sim 200$ –900 GeV) experiments allow one to quantify the scaling violation. The scale-breaking model of Wdowczyk and Wolfendale was proposed to describe the CR data in 1972 [6]. It is, in a sense, a generalization of the Feynman scaling idea introducing one scaling violation parameter.

In Ref. [7] we showed that the light composition suggested by the studies of the anisotropy and the average depth of the shower maximum ( $X_{\text{max}}$ ) does not contradict other results, mainly the measurements of the width of the  $X_{\text{max}}$  distribution available at that time, only if one assume strong Feynman scaling violation. Since that time new results concerning the spread of the shower

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**Fig. 1.** Pseudorapidity distributions shifted by  $\Delta y = \ln(\sqrt{s}/m)$  for ISR, SPS and Tevatron measurements (a), and distributions measured by LHC experiments at energies from 900 GeV to 7 TeV compared with the SPS  $\sqrt{s} = 546$  GeV UA5 result (b).

maxima appeared [2,8] and were analyzed in [9]. The problem of consistency of the average  $X_{max}$  and r.m.s. of  $X_{max}$  measured by two big experiments is the additional point of importance in the studies of high and very energy multiparticle production processes.

The rapidity (pseudorapidity) distributions were measured by the LHC experiments: ALICE [10], CMS [11,12] and ATLAS [13] (the last for  $p_{\perp} > 0.5$  GeV only) in the central rapidity region  $|\eta| \lesssim 2.5$  for c.m.s. energies of 900 GeV, 2.3 TeV and 7 TeV. The narrow range of a rapidity (pseudorapidity) at first sight does not allow one to study important characteristics of very forward particle production. To study the fragmentation region new measurements, especially by much forward detectors (LHCf), are welcome. But, as will be shown below, the existing data can be used to test the scaling violation picture found in the UHECR physics domain.

## 2. Rapidity distribution

Rapidity distributions measured in LHC experiments cover the central region where the produced particles are dynamically separated from the valence quarks of the colliding hadrons. The central rapidity density  $\rho(0) = 1/\sigma (d\sigma/dy)|_{y=0}$  is the variable describing the particle production there. The original Feynman scaling preserves the value of the central rapidity density. The plateau in rapidity is a characteristic feature of the independent jet fragmentation model as well as statistical models with limited transverse momentum phase space. Unfortunately, it has been known long, that such a simple picture does not work.

The phenomenological fit of the  $\rho(0)$  rise made more than twenty years ago in Ref. [14] is still valid. The 900 GeV LHC measurements match well the SPS UA5 result. The systematic discrepancy seen by the CMS detector [11] does not change this general opinion.

### 2.1. Feynman scaling

Feynman scaling [5] can be expressed introducing one universal function  $f_F$  of the variable  $x = p_{\parallel}/p_{max}$  which describes the invariant momentum (longitudinal  $p_{\parallel}$ ) distribution of particles created in the high-energy inelastic (and non-single diffractive) interaction

$$\frac{E}{\sqrt{s}/2} \frac{1}{\sigma} \frac{d^3\sigma}{dx d^2p_{\perp}} = f(x, p_{\perp}, s) = f_F(x, p_{\perp}) \quad (1)$$

where  $\sqrt{s}$  is the interaction c.m.s. energy,  $E$ ,  $p_{\parallel}$  and  $p_{\perp}$  are energy, and longitudinal and transverse momenta of outgoing particles ( $p_{max} \approx \sqrt{s}/2$ ), respectively. Change of variable from Feynman  $x$  to rapidity  $y$  gives

$$\frac{1}{\sigma} \frac{d^3\sigma}{dy d^2p_{\perp}} = f_F(x(y), p_{\perp}) \quad (2)$$

where  $x(y) = \sqrt{p_{\perp}^2 + m^2}/(\sqrt{s}/2) \sinh(y)$ . Using the approximate relation  $\sqrt{p_{\perp}^2 + m^2} \sinh(y) \approx p_{\perp} \sinh(\eta)$  and introducing the very convenient variable: pseudorapidity  $\eta = -\ln \tan(\theta/2)$  we have

$$\frac{1}{\sigma} \frac{d^3\sigma}{d\eta d^2p_{\perp}} = f_F\left(\frac{2p_{\perp}}{\sqrt{s}} \sinh(\eta), p_{\perp}\right). \quad (3)$$

The integration over all  $p_{\perp}$  is obvious when  $p_{\perp}$  and  $p_{\parallel}$  are uncorrelated and the  $p_{\perp}$  distribution does not depend on the interaction energy  $s$ .

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta} = F_F\left(\frac{2\langle p_{\perp} \rangle}{\sqrt{s}} \sinh(\eta)\right). \quad (4)$$

The factor  $\langle p_{\perp} \rangle$  is a constant related to the transverse momentum scale.

We are interested in the extreme forward part of the (pseudo)rapidity distribution – the projectile fragmentation region. It is convenient to move the longitudinal momentum distribution to the anti-laboratory frame ( $\eta \rightarrow \eta'$ ) where the projectile is at rest prior to the collision. This is done shifting the c.m.s. (pseudo)rapidity distribution by  $\Delta y = \ln(\sqrt{s}/m)$

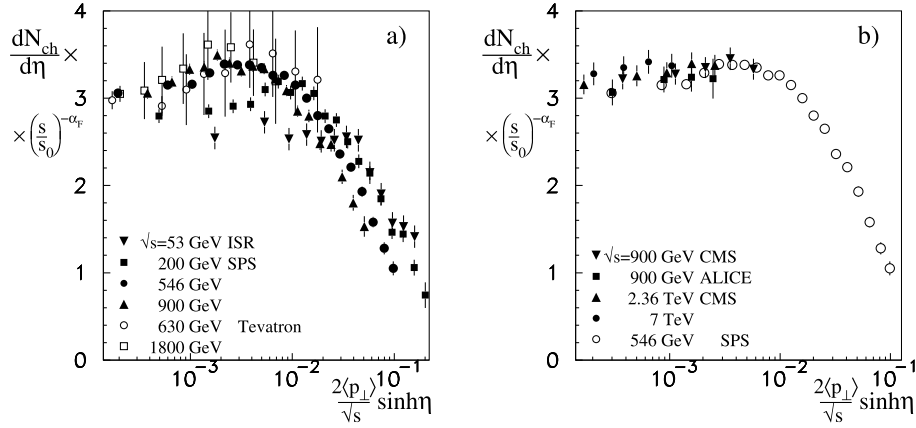
$$\begin{aligned} \sinh(\eta') &= \sinh(\eta - \Delta y) = \sinh(\eta - \ln(\sqrt{s}/m)) \\ &\approx e^{\eta - \ln(\sqrt{s}/m)}/2 = \frac{e^{\eta}}{2} \frac{m}{\sqrt{s}} \approx \frac{m}{\sqrt{s}} \sinh(\eta). \end{aligned} \quad (5)$$

After such transformation a direct comparison of particle production at different values of interaction c.m.s. energy is possible

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta'} \approx F_F\left(\frac{2\langle p_{\perp} \rangle}{m} \sinh(\eta')\right) = F_{\eta}(\eta'). \quad (6)$$

This form of Feynman scaling was tested e.g. in Ref. [14] and it was found that it is valid only very approximately. We can see this in Fig. 1a, where previous millennium data are plotted as a function of the anti-laboratory pseudorapidity. The recent data from CMS [11,12] and ALICE [10] are shown in Fig. 1b.

It is known that Feynman scaling is violated at least by the continuous increase of the central rapidity density, as is easily seen in Fig. 1.



**Fig. 2.** Pseudorapidity distributions shifted and transformed respectively adjusting  $\alpha_F$  for ISR, SPS and Tevatron measurements (a), and distributions measured by LHC experiments at energies from 900 GeV to 7 TeV compared with SPS  $\sqrt{s} = 546$  GeV UA5 result (b).

## 2.2. Feynman scaling violation

The original Feynman scaling implied that the inelasticity of proton–proton interaction, defined as the fraction of incoming energy carried by newly created particle, is universal, the same for all interaction energies. The first observations suggested an attractive value of 0.5. The rise of some characteristics of the interactions (such as, e.g., average  $p_{\perp}$  or central rapidity density as mentioned above) makes the assumption about the constancy of the inelasticity not well justified. Introducing the multiplicative factor proportional to the observed rise of the rapidity plateau to the right-hand side of Eq. (6) we can try to recover a form of scaling. Applying this procedure the simplicity of the original Feynman idea is lost and the next correction for the rise of the average transverse momentum could be introduced here as well. We have used in the present work the average transverse momentum rise of the form  $\langle p_{\perp} \rangle = 0.413 - 0.017 \ln(s) + 0.00143 \ln^2(s)$  shown in Fig. 4 of Ref. [12]. The additional inelasticity control parameter is an index in a power law multiplicative factor. These two modifications lead, according to Eq. (4), to only a slightly more complicated scaling formula

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta} = \left( \frac{s}{s_0} \right)^{\alpha_F} F_F \left( \frac{2\langle p_{\perp} \rangle}{\sqrt{s}} \sinh(\eta) \right). \quad (7)$$

We have used the UA5 data measured at  $\sqrt{s_0} = 546$  GeV c.m.s. energy [14] as a datum. The very accurate measured NSD pseudorapidity distribution have been used as a definition of the universal  $F_F$  function. We adjusted the  $\alpha_F$  parameter value to minimize the discrepancy between Eq. (7) scaling prediction and the distributions of pseudorapidity measured at different energies: from ISR to 7 TeV of LHC. The results are given in Fig. 2.

Values of  $\alpha_F$  increase from  $\sim 0.05$  found for ISR 53 GeV to  $\sim 0.11$  at LHC 7 TeV. The increase is statistically not very significant, at least for the overall inelasticity, which will be discussed later. The accuracy of the data scaling according to Eq. (7) can be estimated with the help of statistical tests. The  $\chi^2$  values for the ISR and SPS are about  $\chi^2/NDF \approx 40/20$ . The systematic uncertainties of the Tevatron and LHC results makes the  $\chi^2/NDF$  smaller but the overall tendency seen in Fig. 2 strongly suggests that the proposed modification of Feynman scaling is not the right solution for the extrapolation of interaction properties to very high interaction energies.

## 2.3. Wdowczyk and Wolfendale scaling

It was shown in Ref. [7] that the almost forty years old modification known as Wdowczyk and Wolfendale (WW) scaling [6] could be still satisfactorily used to scale the interaction properties to the ultra high ( $> 10^{19}$  eV) cosmic ray energies.

The original idea of the WW scaling

$$f(x, p_{\perp}, s) = (s/s_0)^{\alpha} f_{WW}(x(s/s_0)^{\alpha}, p_{\perp}) \quad (8)$$

is an extension of the Feynman fragmentation formula of Eq. (1) (the limit for  $\alpha = 0$ ) with the possibility of getting the ‘thermodynamical limit’ of  $n \sim s^{1/4}$  with  $\alpha = 0.25$ .

The WW model in its version of the mid 1980s has been successfully used for EAS studies around ‘the knee’. Its extension introducing partial inelasticities (energy fraction carried by specific types of particles), and the transverse momentum rise with interaction energy dependencies, as discussed above, gave a better description of the production of different kinds of secondaries. As a result of this improvement the first power-law factor index was released and gave an extra model parameter. This more flexible formula was applied, e.g., in Ref. [14] where the agreement of the WW model predictions and the UA5 measured rapidity distributions was shown. It should be mentioned that the original Wdowczyk and Wolfendale model gave a complete description of the multiparticle production process to be used mainly in EAS studies, so it contains such details as partial inelasticities, transverse momenta, semiinclusive properties etc. The fit shown in Ref. [14] is the effective, average description of inclusive data of rapidity (pseudorapidity) only.

In the present work we explore WW scaling of the form

$$\frac{1}{\sigma} \frac{d\sigma}{d\eta} = \left( \frac{s}{s_0} \right)^{\alpha'} F_{WW} \left( \frac{\langle p_{\perp} \rangle}{\langle p_{\perp}^0 \rangle} \sinh(\eta) \left( \frac{s}{s_0} \right)^{\alpha-1/2} \right), \quad (9)$$

where  $\langle p_{\perp}^0 \rangle$  is the average transverse momentum at the datum interaction energy ( $\sqrt{s_0} = 546$  GeV).

We have adjusted first both  $\alpha$  and  $\alpha'$  parameters independently to get the best scaling performance. The results are given in Fig. 3.

The values of  $\alpha$  and  $\alpha'$  obtained in this way are shown in Fig. 4a. Thin horizontal lines show results from Ref. [14] (solid for  $\alpha$  and dashed for  $\alpha'$ , respectively). The thick solid broken line is the result for  $\alpha$  in our UHECR analysis [7]. It is seen that the predictions from Ref. [7] and the LHC data are consistent. Although the large uncertainties, which are the result of the limited rapidity range as well as possible systematics, do not allow one any stronger conclusions.

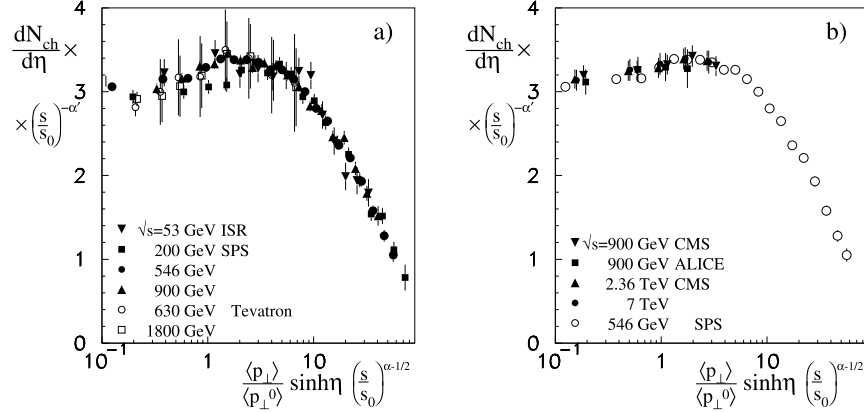


Fig. 3. Wdowczyk and Wolfendale scaling with both parameters  $\alpha$  and  $\alpha'$  adjusted to each experimental data set.

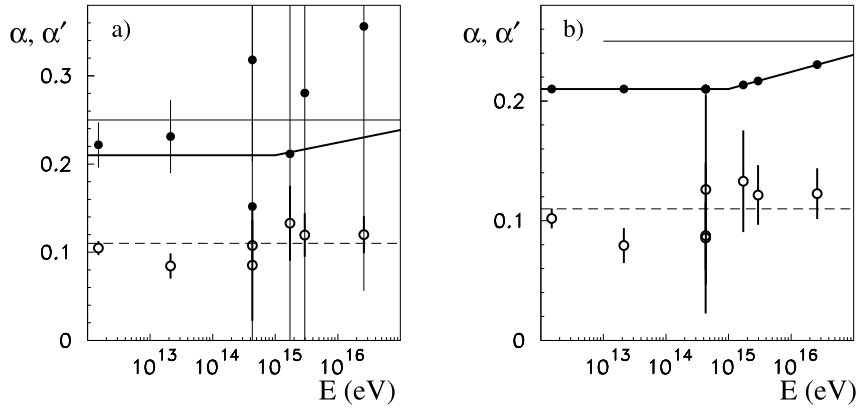


Fig. 4. W&W scaling parameters predictions for  $\alpha$  (solid symbols and solid lines: thin, horizontal – [14], thick, broken – [7]) and for  $\alpha'$  (open symbols and dashed line) adjusted to the data (a), and values of  $\alpha$  taken from the UHECR analysis [7] and only  $\alpha'$  used as a free parameter of the fit (b).

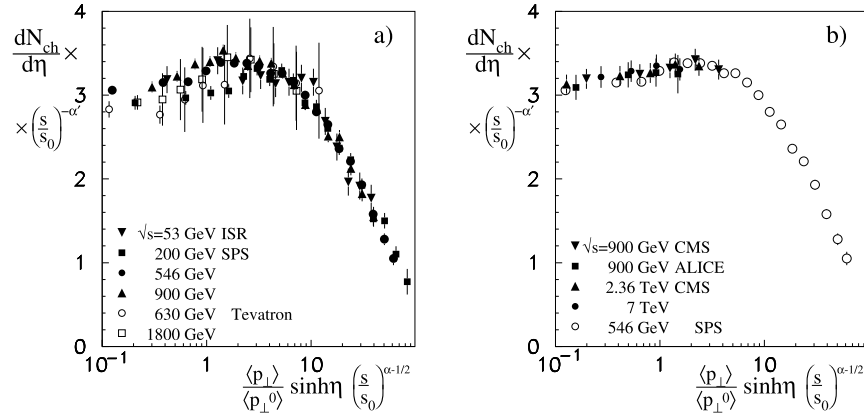


Fig. 5. Wdowczyk and Wolfendale scaling results with  $\alpha$  set to the UHECR analysis data and  $\alpha'$  adjusted to each experimental data set, shown as in Fig. 3.

We can, however, use the UHECR data analysis predictions for the values of  $\alpha$  and test if the results of the fit, with such reduced free parameter space, remains in agreement with the WW scaling. It can be seen in Fig. 5 that the data description is not much worse than the one presented in Fig. 3. The constancy of the  $\alpha'$  suggested by WW original papers and seen in Fig. 4a, still holds as presented in Fig. 4b.

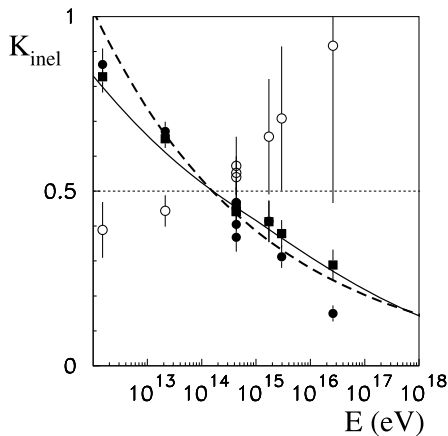
### 3. Inelasticity

In Ref. [7] a quite unexpectedly high energy behaviour of the interaction inelasticity coefficient was found. It was obtained as a

result of the experimental suggestion that the composition of the UHECR is quite light and contains a significant proton fraction. The WW model with strong Feynman scaling violation leads to a continuous decrease of the energy fraction released to the secondaries produced in very high energy interactions. Eq. (9) gives the inelasticity energy dependence

$$K(s) = K_0 \left( \frac{s}{s_0} \right)^{(\alpha' - \alpha)}, \quad (10)$$

while for the modified Feynman scaling formula (7) it is



**Fig. 6.** Inelasticity calculated with WW scaling assumption (filled symbols – circles for both  $\alpha$  and  $\alpha'$  adjusted (Fig. 3) and squares for UHECR inspired  $\alpha$  (Fig. 5), open symbols are for the modified Feynman scaling with  $\alpha_F$  parameter). Solid line shows the UHECR data analysis prediction from Ref. [7]. Dashed line is the inelasticity fit from Ref. [14]. The ‘canonical’ value 0.5 – normalization point at energy of  $10^{14}$  eV – is shown by short dashed line.

$$K(s) = K_0 \left( \frac{s}{s_0} \right)^{\alpha_F}. \quad (11)$$

In Fig. 6 we show the results of our analysis. The open symbols show the fast rise of the inelasticity for the modified Feynman scaling formula. Even if the  $\alpha_F$  follow the lower energy, smaller value, in the UHECR domain saturation is expected. The filled symbols were obtained for WW scaling. The solid line gives the predictions from Ref. [7] obtained using UHECR data. The dashed line is the fit from Ref. [14] of the WW scaling parameters to SPS data. The value of 0.5 is also shown. We normalize the prediction of both models to this value at an energy of  $10^{14}$  eV.

#### 4. Summary

It has been shown that the minimum bias pseudorapidity distributions measured by the LHC experiments can be very well described by the scale-breaking Wdowczyk and Wolfendale formula.

The scaling violation observed for energies up to the SPS  $\sqrt{s} = 900$  GeV and 1800 GeV in Tevatron was upheld recently in the analysis of new UHECR data.

The phenomenological model of Wdowczyk and Wolfendale introduces two model parameters. The value of one of them:  $\alpha$ , was originally found to be equal to 0.13 using interpolation of the  $x_F = p_{\parallel}/p_{\max}$  distributions between  $\sqrt{s} \approx 10$  GeV and ISR energies. Later interpolations including SPS data gave the value of 0.18 and finally the effective value of 0.25 was found in Ref. [14]. The increase of the central rapidity density reported also in Ref. [14] suggests  $\alpha = 2 \times 0.105 = 0.21$ . This value gives the Extensive Air Showers development maximum position  $X_{\max}$  for proton initiated showers not far from that measured [2,3] as is shown in Ref. [7].

The UHECR data suggests a further smooth rise of the scale-breaking parameter. The first measurements at LHC up to 7 TeV c.m.s. energy agree with the trend observed at lower energies and seem to smoothly bridge the accelerator results and these on very high energy interaction of cosmic ray protons. The limited range of measured pseudorapidities does not allow us for a stronger statement. The more forward particle production data is highly welcome.

The rising inelasticity predicted by the (modified) Feynman scaling is obviously in contradiction to the Wdowczyk and Wolfendale scaling that has been shown to describe cosmic ray data.

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