A C T A   U N I V E R S I T A T I S   L O D Z I E N S I S
FOLIA OECONOMICA 24, 1982

Zsuzsa Kapitány*

# DYNAMIC STOCHASTIC SIMULATION OF A SYSTEM CONTROLLED BY STOCK-SIGNALS

The basic economic ideas of this research and the first results on this line were published in a joint paper[1] about autonomous (vegetative) control. The deterministic basic model for this simulation experiment was worked out by J. Kornai and A. Simonovits[2]. A. Simonovits worked out the proofs of the mathematical theorems, J. Kornai contributed the economic interpretation of the assumptions and results[3].

## The deterministic model

We investigated a dynamic system by this model. This deterministic Neumann-economy is controlled by decentralised stock-signals. The real sphere of our deterministic system is described by the following two equations:

$$w(t + 1) = w(t) - Y(t)1 + r(t) \tag{1}$$

* Ass. Professor, Institute of Economics of the Hungarian Academy of Sciences, Budapest, Hungary.

[1] J. K o r n a i, B. M a r t o s, Autonomous functioning of the economic system, "Econometrica" 1973, nr 41, p. 509-528.

[2] J. K o r n a i, A. S i m o n o v i t s, Decentralized control problems in Neumann-economies (in Hungarian), "Szigma" 1975, nr 8, p. 81-89.

[3] The author is very grateful to J. Kornai, B. Martos and A. Simonovits for their ideas, valuable advice and useful remarks.

$$S(t + 1) = S(t) - B < r(t + 1) > - (A - C) < r(t) > + Y(t) \quad (2)$$

where:

$r(t)$ is the n vector of production. The variable $r_j$ is the output of sector j.

$Y(t)$ is the n by n matrix of purchases. The variable $Y_{ij}$ is the quantity of the product i purchased by the sector j.

$w(t)$ is the n vector of the output stocks. The variable $w_j$ is the stock accumulated by sector j from its own product in its own inventory.

$S(t)$ is the n by n matrix of the stock input stocks. The variable $s_{ij}$ is the part of the total input stock above the technologically necessary amount.

The total input stock is composed of two parts, the technological input stock and the slack input stock. The sum of the output stocks and the slack input stocks of the economy is called: buffer stock.

A is the n by n matrix of the current input coefficients. This matrix is well known from the static and dynamic Leontief models.

B is the n by n matrix of the technologically indispensable input stock coefficients.

C is the n by n matrix of technological input stock coefficients without scrapping.

The ensemble of A, B, C matrices is called the real structure of the system. The real structure is constant in time.

In the control sphere normative control is assumed. There is a normative path of the system, a Neumann path plays this role. We denote the corresponding variables on the normative path by the same letters by an asterisk. This path is described by the following forms:

$$r^*(t) = r_o \lambda_o^t$$

$$Y^*(t) = Y_o \lambda_o^t$$

$$w^*(t) = w_o \lambda_o^t \quad (3)$$

$$S^*(t) = S_o \lambda_o^t$$

where $\lambda_o$ is the Neumann growth coefficient, $\lambda_o > 1$.

There is a unique Neumann-path which satisfies equations (1)-(2) but only under the following conditions: Let us denote the matrix of the norms of the slack input stock per output by F and the vector of the norms of the output stock per production by g.

Let us denote by H the matrix of the norms of buffer stocks: $H = F + \langle g \rangle$. The Neumann-path (3) exists and is unique if $\lambda_o$ and $r_o$ are solutions of the following eigenvalue-eigenvector problem:

$$\lambda_o (B + H) r_o = (I - A + C + H) r_o$$

such that $\lambda_o > 1$ and $r_o > 0$, moreover

$$w_o = \langle g \rangle r_o$$

$$S_o = F \langle r_o \rangle$$

$$Y_o = [(\lambda_o - 1) F + A + \lambda_o B + C] \langle r_o \rangle .$$

The normative Neumann-path is determined by F and $\langle g \rangle$ if the real sphere is given. It is proved that comparing the economies 1 and 2 which are fully identical except for the norms of the buffer stocks $h_{ij}^{(1)} = h_{ij}^{(2)}$ for every $(i, j)$ and at least one component is definitely larger in economy 1 than in economy 2, than the growth of economy 1 is smaller than that of 2: $\lambda_o^{(1)} < \lambda^{(2)}$. It means in the deterministic model that the increase of the buffer stock norms slows down the growth of the economy.

The control of the deterministic system is described in the following two equations:

$$r(t) = r^*(t) - d \circledast [w(t) - w^*(t)] \tag{4}$$

$$Y(t) = Y^*(t) - E \circledast [S(t) - S^*(t)] \tag{5}$$

where the logical product of two matrices by elements is denoted by symbol $\circledast$,

d is the vector of adjustment speeds of production,

E is the matrix of adjustment speeds of purchases.

These equations relate the deviations between the actual and normative values of the state variables to the deviations of the control variables from their normative values. The system (1)-(5) is viable[4] if none of the variables is negative and in each period at least one production variable is positive. We also use this definition of viability in our stochastic model. Both the deterministic system and the stochastic system is calculated recursively.

## The Stochastic Model

The stochastic variant differs from the deterministic system only in control equations (4)-(5). We assume the stochastic disturbance between the actual and determined values of production and purchases. Let us denote the actual values by $\bar{r}$ and $\bar{Y}$:

$$\bar{r}_i(t) = p_i(t)\, r_i\,(t) \qquad i = 1, 2, \ldots, n$$

$$\bar{Y}_{ij}t = \pi_{ij}(t)\, Y_{ij}(t) \qquad i, j = 1, 2, \ldots, n \tag{5}$$

where $p_i\,(t)$ and $\pi_{ij}(t)$ are random numbers expressing the stochastic disturbances caused by the irregular behavior of the decision makers.

The disturbance is multiplicative. The values of the random numbers $p_i\,(t)$ and $\pi_{ij}(t)$ are distributed uniformly on a given interval around 1.

## The Problems of the Simulation Experiment

Let us now turn to the buffer stock and growth rate. It is proved that in the deterministic model the increase of the buffer stock norms slows down the growth of the economy.

In the real life the increasing of the buffer stock norms

---

[4] For the same concept Martos uses the term "practicability" in his paper delivered to this conference.

might be useful because it may help to overcome the unexpected disturbances of the production, may make adaption smoother etc.

We tried to investigate these questions experimentally. The first question of the experiment was: How does the viability of the system vary by increasing the buffer stock per production norms? Can we improve the viability of the system in this way? This question is relevant only if the model is stochastic. But we were unable to investigate the stochastic model by analytical method. Hence we used computer simulation and the Monte Carlo technique to study this problem. The viability does not depend on the buffer stock per production only, therefore the second question was the following: How does the viability of the system vary by increasing the random disturbances? In the experiments the number of periods, the investigated time also varied. The third question was: How does the viability depend on the number of periods?

## Factors of the Simulation

Let us denote the number of period by T. The $T(n^2 + n)$ random numbers are uniformly distributed on the interval $[1 - \alpha; \quad 1 + \alpha]$ and independent of each other. $\alpha$ and T are factors of the simulation.

Let us call it a run when we calculate one path of our model variables with T and $\alpha$ given.

An experiment consists of several runs, let their number be z. In there runs the same $\alpha$ and T but different series of the random variables are used. z number of runs in an experiment is also a factor of the simulation. There is one more factor of the simulation, let us denote it by $\beta$. $\beta$ means the ratio of the elements of matrices F and B, i.e. the ratio of slack input stock norms to technological input coefficients. The simulation consists of several series of experiments in which the values of the factors, $\alpha$, $\beta$, z and T are different. The starting values of the factors and the other model parameters were chosen in such a way that the economic system simulated by our model should be realistic. For example: in one experiment-series the factor $\beta$ is about 0.4, so that the growth coefficient is about 1.05.

## The Response of the Simulation

A run is viable if none of the model variable is negative during T periods (see fig. 1). A run is non-viable if there is a variable in a period which is negative. Let m be the number of the viable runs among z runs. Further the probability of the viability of the system is called viability. Theoretically the viability is a binomially distributed stochastic variable. We estimate the value of this variable by m/z. This variable depends on the factors $\alpha$, $\beta$, z and T and m/z is the response of the simulation.

## Results

At any level of the buffer stock per production norms an increase of the random disturbances monotonely decreases the viability but at different speeds (see tab. 1). The path of the stochastic model depends approximately linearly on the random disturbances. We expected that increasing the buffer stock per production norms the viability would increase (namely, if, $\beta_1 < \beta_2$ then $m_1/z < m_2/z$). This hypothesis however is correct in an interval of "small norms" only and fails for larger values of the norms. If the random disturbances are small the viability is relatively large. In this case the increase of the buffer stocks is superfluous. If the random disturbances are moderately strong both too small and too large norms would be harmful because the viability of the system is small in both cases. There is a medium value of the norms at which the viability of the system is maximum. At large random disturbances there is no norm which could help to fight these disturbances. The viability is very small.

In the interval of "small" norms with the increasing of the value of $\beta$ the viability of the system increases steeply. Beyond this interval there is a characteristic one where the system is able to survive moderately strong random disturbances. If the value of $\beta$ increases further the system can survive small disturbances only. Increasing the buffer stock per production norms the capability of the system to survive the disturbances after

Table 1

The dependence of the viability on $\alpha$, $\beta$ and $\lambda_o$

$z = 300$; $T = 100$

| $\beta$ | 0.02 | 0.04 | 0.08 | 0.12 | 0.16 | 0.2 | 0.24 | 0.4 | 1. | 4. | 8. | 20. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a \ $\lambda_o^{-1}$ | 0.079 | 0.078 | 0.075 | 0.072 | 0.070 | 0.068 | 0.066 | 0.058 | 0.042 | 0.017 | 0.007 | 0.004 |
| 0.01 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| – | – | – | – | – | – | – | – | – | – | – | – | – |
| 0.02 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.025 | 0 | 0 | 0,96 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.03 | 0 | 0 | 0.36 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0.035 | 0 | 0 | 0.04 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 | 0.96 |
| 0.04 | 0 | 0 | 0 | 0.68 | 0.92 | 0.92 | 0.92 | 0.92 | 0.92 | 0.88 | 0.80 | 0.80 |
| 0.045 | 0 | 0 | 0 | 0.28 | 0.88 | 0.88 | 0.88 | 0.88 | 0.76 | 0.60 | 0.48 | 0.40 |
| 0.05 | 0 | 0 | 0 | 0.04 | 0.56 | 0.56 | 0.52 | 0.44 | 0.36 | 0.24 | 0.20 | 0.20 |
| 0.055 | 0 | 0 | 0 | 0 | 0.12 | 0.28 | 0.28 | 0.20 | 0.16 | 0 | 0 | 0 |
| 0.06 | 0 | 0 | 0 | 0 | 0 | 0.04 | 0.04 | 0.04 | 0 | 0 | 0 | 0 |

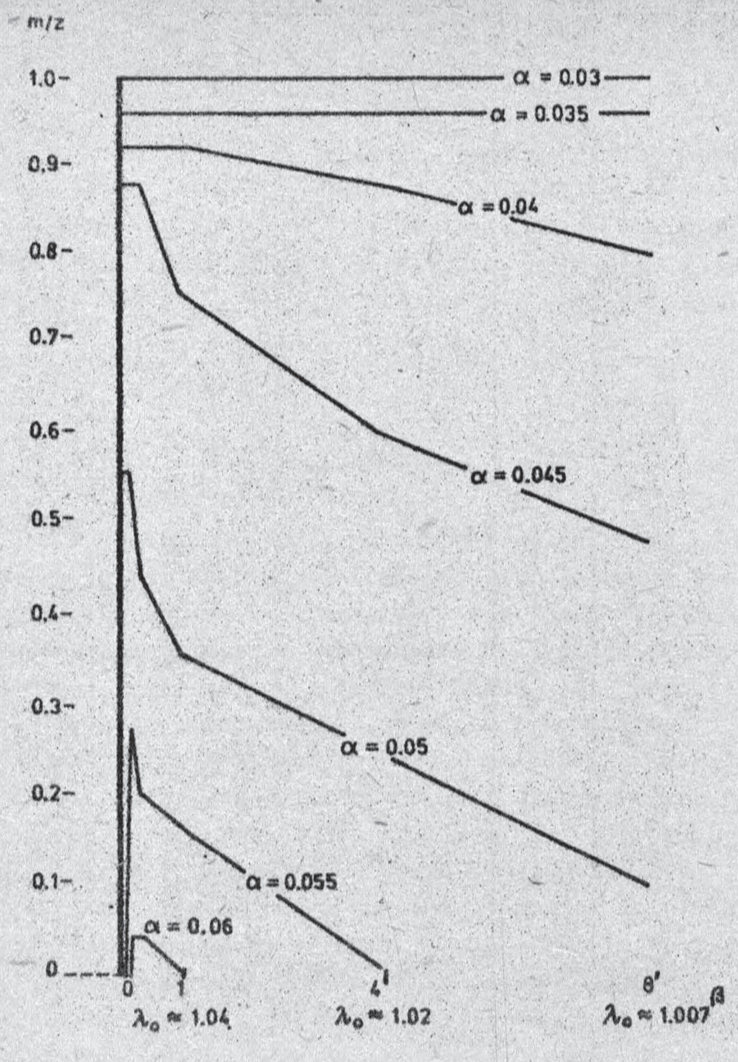


Fig. 1. The Dependence of the Viability on $\beta$, at Different Levels of $\alpha$; z = 300, T = 100

a sudden increase takes up its maximum value and over this norm the viability decreases again. It is interesting that the $\beta$ value which gives the maximum safety seems not to depend on the size of the disturbances.

We try to explain these findings by means of $\lambda_o$, the growth coefficient of the normative path. In our experiment the value of $\lambda_o$ is determined by norm F only and $\lambda_o$ is very close to $\lambda$, the growth coefficient of the system. We do not make a considerable bias if we use $\lambda_0$ instead of $\lambda$.

It was proved in the basic deterministic model that increasing the buffer stock per production norms $\lambda_0$ decreases (see tab.1). At a series of the random disturbances where $\alpha \ll \lambda_0 - 1$, the viability of the system is very large and does not depend on the value of $\beta$. On the other hand if $\alpha \approx \lambda_0 - 1$, the viability of the system is uncertain. Finally if $\alpha \gg \lambda_0 - 1$, the viability of the system decreases rapidly.

The conclusions of the simulation are the following: the viability of the system depends on the buffer stock per production norms in two ways. The increase of the buffer stock per productions norms rises the safety of the system (at the small $\beta$ values) and decreases the growth rate (at the large $\beta$ values). Sometimes the increase of the stock norms is not useful. We refrain from drawing numerical conclusion, rather restrict the conclusions of the experiments to theoretical propositions: there are qualitatively distinct intervals of the factors. There are values of the random disturbances and the stock norms at which the behavior of the system changes qualitatively.

Zsuzsa Kapitány

## DYNAMICZNA STOCHASTYCZNA SYMULACJA SYSTEMU STEROWANEGO PRZEZ SYGNAŁY O ZAPASACH

Wiadomo, że w modelu deterministycznym wzrost normatywnych zapasów powoduje spadek tempa wzrostu gospodarki. W praktyce podniesienie norm zapasów może mieć jednak sens, o ile umożliwia pokonanie nieoczekiwanych trudności i zakłóceń w produkcji, ułatwia adaptację systemu do zmieniających się warunków otoczenia itd.

Prezentowana praca, posługując się eksperymentem symulacyjnym, weryfikuje powyższe tezy, rozpatruje wpływ zwiększenia zapasów "buforowych" na żywotność systemu w modelach deterministycznym i stochastycznym.