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ON CRITICAL VALUES OF GOLDFELD-QUANDT PEAK TEST IN A LINEAR TREND CASE

1. INTRODUCTION

Consider the econometric linear model

$$y = X\alpha + \varepsilon \tag{1}$$

where the possibility of the heteroscedastity of the random term is admitted, and other classical assumptions are met[1]. Therefore it is postulated that the observation n x (k + 1) matrix **X** is non-stochastic with maximal rank and the variance of the random term is

$$V(\varepsilon) = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & 0 \dots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ 0 & \dots & 0 & \sigma_n^2 \end{bmatrix}. \tag{2}$$

Let vector **e** of elements $e_1, e_2, \dots, e_n$ be the vector of OLS residuals. Goldfeld and Quandt (1965) proposed a very simple "peak test" known also as non-parametric Goldfeld-Quandt test to verify the hypothesis on monotonical heterosce-

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[1] Our considerations in this paper are independent from normality assumption of the random term.

dastity of the random term of model (1). We say that the residual e_t forms a "peak" if

$$|e_u| < |e_t| \qquad u = 1, 2, \ldots, t - 1 \tag{3}$$

for $u = 1, 2, \ldots, t - 1$. The test statistic is the number of peaks $G = G(e)$ in the sample. If many peaks are observed then this is against the null hypothesis

$$H_0\colon \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_n^2 \tag{4}$$

and for the alternative

$$H_1\colon \sigma_1^2 \le \sigma_2^2 \le \ldots \le \sigma_n^2 . \tag{5}$$

Therefore a right-hand-side critical region must be constructed against (5). With the alternative hypothesis

$$H_0\colon \sigma_1^2 \ge \sigma_2^2 \ge \ldots \ge \sigma_n^2$$

we get analogous distribution changing condition (3) into $|e_u| > |e_t|$ for $u = 1, 2, \ldots, t - 1$ (in such a case the term "peak" is obviously unsuitable) and the critical region will be right-hand-side as well. Because of the shape of the distribution of G, the left-hand-side critical region is not recommended.

Let us denote by $M(n, g)$ the number of permutations of residuals e_t for which g peaks are observed i.e. the number of such permutation matrices for which

$$G(Pe) = g.$$

We have

$$M(n, n - 1) = 1 \tag{6}$$

for $n = 2, 3, \ldots, n$, because only one permutation orders the vector e so as the elements of the vector $v = Pe$ fulfill the inequality

$$|v_1| \le |v_2| \le \ldots \le |v_n|$$

and

$$M(n, 0) = 1 \tag{7}$$

because there is no peak if

$$|v_1| \ge |v_1| \ge \ldots \ge |v_n|.$$

For a given g $(2 \le g \le n - 2)$ we get

$$M(n, g) = (n - 1)M(n - 1, g) + M(n - 1, g - 1) \tag{8}$$

because the element $|v_n|$ forms a peak if and only if it is the biggest one.

Equations (6)-(8) define a recursive formula for the function M. When we assume that the probability of every permutation **Pe** is the same, the probability that there are g peaks in an n-element sample can be expressed as follows

$$P(n, g) = \frac{1}{n} M(n, g) \tag{9}$$

Cumulated values

$$F(n, g) = \sum_{h=0}^{g} P(n, h) \tag{10}$$

for some n (n = 5, 10, 15, ..., 60) were given by Goldfeld and Quandt (1965). Tables for n = 20, 40, 60, 80, 100, 150, 200, 250, 300, 350, 400, 450, 500 can be found in the book by Goldfeld and Quandt (1972, p. 121).

2. TABLES AND APPROXIMATION FORMULAE

However, Goldfeld´s and Quandt´s tables are quite seldom, their practical application demands interpolation. This paper presents largely extended tables of the distribution of the statistic G (see Table 1).

Of course, the use of the tables in computer calculations is very inconvenient. Approximation formulae of various kinds according to which the approximate critical values of the applied test can be counted are more appropriate. Such formulae for the so called interpolated quantiles which are closely connected with randomized tests are given. The G statistic is discrete. Thus, for a given significance level α it is in general impossible to find such a critical value $g(n, \alpha)$ that

$$P(G \geq g(n, \alpha)) = \alpha.$$

Whereas taking

$$g(n, \alpha) = \min\{g: P(G \geq g) > \alpha\} \tag{11}$$

we obtain a test whose size is smaller than α.

Let us accept (11) and define the following test procedure

1) if $G \geq g(n, \alpha)$ H_0 is rejected,

2) if $G < g(n, \alpha) - 1$ H_0 is accepted,

Table 1

Peak test statistic distribution

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	.5000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
3	.3333	.8333	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
4	.2500	.7083	.9583	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
5	.2000	.6167	.9083	.9917	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	.1667	.5472	.8597	.9778	.9986	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	.1429	.4929	.8151	.9609	.9956	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
8	.1250	.4491	.7748	.9427	.9913	.9993	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
9	.1111	.4131	.7386	.9240	.9859	.9984	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10	.1000	.3829	.7061	.9055	.9797	.9971	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
11	.0909	.3572	.6767	.8874	.9730	.9956	.9995	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
12	.0833	.3350	.6501	.8698	.9658	.9937	.9992	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
13	.0769	.3156	.6258	.8529	.9584	.9915	.9988	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
14	.0714	.2986	.6037	.8367	.9509	.9892	.9982	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
15	.0667	.2834	.5833	.8211	.9433	.9866	.9976	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
16	.0625	.2699	.5646	.8063	.9357	.9839	.9969	.9996	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
17	.0588	.2577	.5472	.7921	.9280	.9811	.9962	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
18	.0556	.2466	.5312	.7785	.9205	.9781	.9953	.9992	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
19	.0526	.2366	.5162	.7654	.9130	.9751	.9944	.9990	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
20	.0500	.2274	.5022	.7530	.9056	.9720	.9935	.9988	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
21	.0476	.2189	.4891	.7410	.8984	.9688	.9924	.9985	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
22	.0455	.2112	.4768	.7296	.8912	.9656	.9914	.9983	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
23	.0435	.2039	.4653	.7186	.8842	.9624	.9903	.9980	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
24	.0417	.1973	.4544	.7080	.8773	.9591	.9891	.9976	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
25	.0400	.1910	.4441	.6979	.8705	.9559	.9879	.9973	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
26	.0385	.1852	.4344	.6881	.8639	.9526	.9867	.9969	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
27	.0370	.1798	.4252	.6787	.8574	.9493	.9854	.9966	.9993	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
28	.0357	.1747	.4164	.6697	.8510	.9460	.9841	.9962	.9992	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
29	.0345	.1699	.4081	.6610	.8447	.9427	.9828	.9957	.9991	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	.0333	.1654	.4001	.6525	.8386	.9395	.9815	.9953	.9990	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
31	.0323	.1611	.3925	.6444	.8326	.9362	.9801	.9949	.9989	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
32	.0312	.1571	.3853	.6365	.8267	.9330	.9787	.9944	.9988	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
33	.0303	.1533	.3784	.6289	.8210	.9298	.9773	.9939	.9986	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
34	.0294	.1497	.3718	.6215	.8153	.9266	.9759	.9934	.9985	.9997	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
35	.0286	.1462	.3654	.6144	.8098	.9234	.9745	.9929	.9984	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
36	.0278	.1430	.3593	.6075	.8044	.9202	.9731	.9924	.9982	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
37	.0270	.1399	.3535	.6008	.7990	.9171	.9717	.9919	.9980	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
38	.0263	.1369	.3479	.5943	.7938	.9140	.9702	.9914	.9979	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
39	.0256	.1340	.3425	.5879	.7887	.9109	.9688	.9908	.9977	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
40	.0250	.1313	.3373	.5818	.7837	.9078	.9674	.9903	.9975	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
41	.0244	.1287	.3322	.5758	.7788	.9048	.9659	.9897	.9974	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
42	.0238	.1263	.3274	.5700	.7739	.9018	.9645	.9892	.9972	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
43	.0233	.1239	.3227	.5644	.7692	.8988	.9630	.9886	.9970	.9993	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
44	.0227	.1216	.3182	.5589	.7645	.8959	.9615	.9880	.9968	.9993	.9999	1.000	1.000	1.000	1.000	1.000	1.000	1.000
45	.0222	.1194	.3138	.5536	.7600	.8930	.9601	.9874	.9966	.9992	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
46	.0217	.1173	.3096	.5483	.7555	.8901	.9586	.9868	.9964	.9992	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
47	.0213	.1152	.3055	.5433	.7511	.8872	.9572	.9862	.9962	.9991	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
48	.0208	.1133	.3015	.5383	.7467	.8844	.9557	.9856	.9960	.9990	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
49	.0204	.1114	.2977	.5335	.7425	.8816	.9542	.9850	.9958	.9990	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	.0200	.1096	.2940	.5288	.7383	.8788	.9528	.9844	.9956	.9989	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
51	.0196	.1078	.2904	.5242	.7342	.8760	.9513	.9838	.9954	.9988	.9998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
52	.0192	.1061	.2868	.5197	.7302	.8733	.9499	.9831	.9951	.9988	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000
53	.0189	.1045	.2834	.5153	.7262	.8706	.9485	.9825	.9949	.9987	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000
54	.0185	.1029	.2801	.5110	.7223	.8679	.9470	.9819	.9947	.9986	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000
55	.0182	.1014	.2769	.5068	.7184	.8653	.9456	.9813	.9944	.9986	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000
56	.0179	.0999	.2738	.5027	.7147	.8627	.9441	.9806	.9942	.9985	.9997	.9999	1.000	1.000	1.000	1.000	1.000	1.000
57	.0175	.0984	.2707	.4987	.7109	.8601	.9427	.9800	.9940	.9984	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000
58	.0172	.0971	.2677	.4947	.7073	.8575	.9413	.9793	.9937	.9983	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000
59	.0169	.0957	.2649	.4909	.7037	.8549	.9399	.9787	.9935	.9983	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000
60	.0167	.0944	.2620	.4871	.7001	.8524	.9384	.9780	.9932	.9982	.9996	.9999	1.000	1.000	1.000	1.000	1.000	1.000
61	.0164	.0931	.2593	.4834	.6966	.8499	.9370	.9774	.9930	.9981	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000
62	.0161	.0919	.2566	.4798	.6932	.8475	.9356	.9767	.9927	.9980	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000
63	.0159	.0907	.2540	.4763	.6898	.8450	.9342	.9761	.9925	.9979	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000
64	.0156	.0895	.2514	.4728	.6865	.8426	.9328	.9754	.9922	.9979	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000
65	.0154	.0884	.2489	.4694	.6832	.8402	.9315	.9748	.9920	.9978	.9995	.9999	1.000	1.000	1.000	1.000	1.000	1.000
66	.0152	.0873	.2465	.4660	.6799	.8378	.9301	.9741	.9917	.9977	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000
67	.0149	.0862	.2441	.4628	.6768	.8354	.9287	.9735	.9914	.9976	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000
68	.0147	.0851	.2418	.4596	.6736	.8331	.9273	.9728	.9912	.9975	.9994	.9999	1.000	1.000	1.000	1.000	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
69	.0145	.0841	.2395	.4564	.6705	.8308	.9260	.9721	.9909	.9974	.9993	.9999	1.000	1.000	1.000	1.000	1.000	1.000
70	.0143	.0831	.2373	.4533	.6674	.8285	.9246	.9715	.9906	.9973	.9993	.9998	1.000	1.000	1.000	1.000	1.000	1.000
71	.0141	.0822	.2351	.4503	.6644	.8262	.9232	.9708	.9904	.9972	.9993	.9998	1.000	1.000	1.000	1.000	1.000	1.000
72	.0139	.0812	.2330	.4473	.6615	.8240	.9219	.9702	.9901	.9971	.9993	.9998	1.000	1.000	1.000	1.000	1.000	1.000
73	.0137	.0803	.2309	.4443	.6585	.8218	.9206	.9695	.9898	.9970	.9992	.9998	1.000	1.000	1.000	1.000	1.000	1.000
74	.0135	.0794	.2289	.4415	.6556	.8196	.9192	.9688	.9896	.9969	.9992	.9998	1.000	1.000	1.000	1.000	1.000	1.000
75	.0133	.0785	.2269	.4386	.6528	.8174	.9179	.9682	.9893	.9968	.9992	.9998	1.000	1.000	1.000	1.000	1.000	1.000
76	.0132	.0776	.2250	.4358	.6500	.8152	.9166	.9675	.9890	.9967	.9991	.9998	1.000	1.000	1.000	1.000	1.000	1.000
77	.0130	.0768	.2230	.4331	.6472	.8131	.9153	.9669	.9887	.9966	.9991	.9998	1.000	1.000	1.000	1.000	1.000	1.000
78	.0128	.0760	.2212	.4304	.6444	.8109	.9139	.9662	.9884	.9965	.9991	.9998	1.000	1.000	1.000	1.000	1.000	1.000
79	.0127	.0752	.2193	.4278	.6417	.8088	.9126	.9655	.9882	.9964	.9990	.9998	1.000	1.000	1.000	1.000	1.000	1.000
80	.0125	.0744	.2175	.4252	.6390	.8067	.9113	.9649	.9879	.9963	.9990	.9998	.9999	1.000	1.000	1.000	1.000	1.000
81	.0123	.0736	.2158	.4226	.6364	.8047	.9100	.9642	.9876	.9962	.9990	.9998	.9999	1.000	1.000	1.000	1.000	1.000
82	.0122	.0729	.2140	.4201	.6338	.8026	.9088	.9636	.9873	.9961	.9989	.9997	.9999	1.000	1.000	1.000	1.000	1.000
83	.0120	.0722	.2123	.4176	.6312	.8006	.9075	.9629	.9870	.9960	.9989	.9997	.9999	1.000	1.000	1.000	1.000	1.000
84	.0119	.0715	.2107	.4151	.6287	.7986	.9062	.9622	.9867	.9959	.9989	.9997	.9999	1.000	1.000	1.000	1.000	1.000
85	.0118	.0708	.2090	.4127	.6262	.7966	.9049	.9616	.9864	.9958	.9988	.9997	.9999	1.000	1.000	1.000	1.000	1.000
86	.0116	.0701	.2074	.4104	.6237	.7946	.9037	.9609	.9862	.9957	.9988	.9997	.9999	1.000	1.000	1.000	1.000	1.000
87	.0115	.0694	.2058	.4080	.6212	.7926	.9024	.9603	.9859	.9956	.9988	.9997	.9999	1.000	1.000	1.000	1.000	1.000
88	.0114	.0687	.2043	.4057	.6188	.7907	.9012	.9596	.9856	.9955	.9987	.9997	.9999	1.000	1.000	1.000	1.000	1.000
89	.0112	.0681	.2028	.4035	.6164	.7887	.8999	.9589	.9853	.9954	.9987	.9997	.9999	1.000	1.000	1.000	1.000	1.000
90	.0111	.0675	.2013	.4012	.6140	.7868	.8987	.9583	.9850	.9952	.9987	.9997	.9999	1.000	1.000	1.000	1.000	1.000
91	.0110	.0668	.1998	.3990	.6117	.7849	.8975	.9576	.9847	.9951	.9986	.9997	.9999	1.000	1.000	1.000	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
92	.0109	.0662	.1984	.3969	.6094	.7831	.8963	.9570	.9844	.9950	.9986	.9996	.9999	1.000	1.000	1.000	1.000	1.000
93	.0108	.0656	.1969	.3947	.6071	.7812	.8950	.9563	.9841	.9949	.9985	.9996	.9999	1.000	1.000	1.000	1.000	1.000
94	.0106	.0651	.1955	.3926	.6049	.7793	.8938	.9557	.9838	.9948	.9985	.9996	.9999	1.000	1.000	1.000	1.000	1.000
95	.0105	.0645	.1942	.3906	.6026	.7775	.8926	.9550	.9835	.9947	.9985	.9996	.9999	1.000	1.000	1.000	1.000	1.000
96	.0104	.0639	.1928	.3885	.6004	.7757	.8914	.9544	.9832	.9946	.9984	.9996	.9999	1.000	1.000	1.000	1.000	1.000
97	.0103	.0634	.1915	.3865	.5982	.7739	.8902	.9537	.9829	.9944	.9984	.9996	.9999	1.000	1.000	1.000	1.000	1.000
98	.0102	.0628	.1902	.3845	.5961	.7721	.8890	.9531	.9826	.9943	.9984	.9996	.9999	1.000	1.000	1.000	1.000	1.000
99	.0101	.0623	.1889	.3825	.5939	.7703	.8879	.9524	.9823	.9942	.9983	.9996	.9999	1.000	1.000	1.000	1.000	1.000
100	.0100	.0618	.1876	.3806	.5918	.7685	.8867	.9518	.9820	.9941	.9983	.9995	.9999	1.000	1.000	1.000	1.000	1.000
110	.0091	.0570	.1760	.3626	.5719	.7516	.8752	.9454	.9790	.9929	.9978	.9994	.9999	1.000	1.000	1.000	1.000	1.000
120	.0083	.0530	.1659	.3466	.5538	.7360	.8643	.9391	.9760	.9916	.9974	.9993	.9998	1.000	1.000	1.000	1.000	1.000
130	.0077	.0495	.1571	.3324	.5374	.7214	.8540	.9330	.9729	.9903	.9969	.9991	.9998	.9999	1.000	1.000	1.000	1.000
140	.0071	.0465	.1493	.3196	.5223	.7077	.8440	.9271	.9699	.9890	.9964	.9989	.9997	.9999	1.000	1.000	1.000	1.000
150	.0067	.0439	.1424	.3081	.5085	.6950	.8346	.9212	.9669	.9876	.9958	.9987	.9997	.9999	1.000	1.000	1.000	1.000
160	.0062	.0416	.1362	.2975	.4956	.6829	.8255	.9156	.9638	.9862	.9953	.9985	.9996	.9999	1.000	1.000	1.000	1.000
170	.0059	.0395	.1305	.2879	.4837	.6716	.8168	.9100	.9609	.9848	.9947	.9983	.9995	.9999	1.000	1.000	1.000	1.000
180	.0056	.0376	.1254	.2790	.4726	.6609	.8085	.9046	.9579	.9834	.9941	.9981	.9995	.9999	1.000	1.000	1.000	1.000
190	.0053	.0359	.1208	.2708	.4622	.6507	.8005	.8994	.9550	.9820	.9935	.9979	.9994	.9998	1.000	1.000	1.000	1.000
200	.0050	.0344	.1165	.2632	.4525	.6411	.7928	.8943	.9521	.9806	.9929	.9977	.9993	.9998	1.000	1.000	1.000	1.000
210	.0048	.0330	.1126	.2562	.4433	.6319	.7853	.8893	.9492	.9792	.9923	.9974	.9992	.9998	.9999	1.000	1.000	1.000
220	.0045	.0317	.1089	.2496	.4347	.6231	.7782	.8844	.9464	.9777	.9917	.9972	.9991	.9998	.9999	1.000	1.000	1.000
230	.0043	.0305	.1055	.2434	.4265	.6148	.7713	.8796	.9436	.9763	.9910	.9969	.9990	.9997	.9999	1.000	1.000	1.000
240	.0042	.0294	.1024	.2376	.4188	.6068	.7646	.8750	.9408	.9749	.9904	.9967	.9989	.9997	.9999	1.000	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
250	.0040	.0284	.0994	.2321	.4114	.5991	.7582	.8705	.9381	.9735	.9898	.9964	.9989	.9997	.9999	1.000	1.000	1.000
260	.0038	.0274	.0967	.2270	.4044	.5918	.7519	.8660	.9355	.9721	.9891	.9961	.9988	.9996	.9999	1.000	1.000	1.000
270	.0037	.0266	.0941	.2221	.3978	.5847	.7459	.8617	.9328	.9707	.9885	.9959	.9987	.9996	.9999	1.000	1.000	1.000
280	.0036	.0258	.0917	.2175	.3914	.5780	.7400	.8575	.9302	.9693	.9878	.9956	.9985	.9996	.9999	1.000	1.000	1.000
290	.0034	.0250	.0894	.2131	.3854	.5714	.7343	.8533	.9276	.9679	.9871	.9953	.9984	.9995	.9999	1.000	1.000	1.000
300	.0033	.0243	.0872	.2090	.3796	.5651	.7288	.8493	.9251	.9665	.9865	.9950	.9983	.9995	.9999	1.000	1.000	1.000
310	.0032	.0236	.0852	.2050	.3740	.5591	.7234	.8453	.9226	.9652	.9858	.9947	.9982	.9994	.9998	1.000	1.000	1.000
320	.0031	.0229	.0833	.2012	.3687	.5532	.7182	.8414	.9201	.9638	.9852	.9945	.9981	.9994	.9998	1.000	1.000	1.000
330	.0030	.0223	.0814	.1976	.3635	.5476	.7131	.8376	.9177	.9624	.9845	.9942	.9980	.9994	.9998	1.000	1.000	1.000
340	.0029	.0218	.0797	.1942	.3586	.5421	.7082	.8339	.9153	.9611	.9838	.9939	.9979	.9993	.9998	.9999	1.000	1.000
350	.0029	.0212	.0780	.1909	.3539	.5368	.7034	.8303	.9129	.9597	.9832	.9936	.9978	.9993	.9998	.9999	1.000	1.000
360	.0028	.0207	.0764	.1877	.3493	.5316	.6987	.8267	.9106	.9584	.9825	.9933	.9976	.9992	.9998	.9999	1.000	1.000
370	.0027	.0202	.0749	.1847	.3449	.5267	.6941	.8232	.9083	.9571	.9818	.9930	.9975	.9992	.9998	.9999	1.000	1.000
380	.0026	.0198	.0735	.1818	.3406	.5218	.6896	.8197	.9060	.9558	.9812	.9927	.9974	.9991	.9997	.9999	1.000	1.000
390	.0026	.0193	.0721	.1790	.3365	.5171	.6853	.8163	.9037	.9545	.9805	.9924	.9973	.9991	.9997	.9999	1.000	1.000
400	.0025	.0189	.0708	.1763	.3326	.5126	.6810	.8130	.9015	.9532	.9798	.9921	.9971	.9991	.9997	.9999	1.000	1.000
410	.0024	.0185	.0695	.1737	.3287	.5081	.6769	.8097	.8993	.9519	.9792	.9918	.9970	.9990	.9997	.9999	1.000	1.000
420	.0024	.0181	.0683	.1712	.3250	.5038	.6728	.8065	.8971	.9506	.9785	.9915	.9969	.9990	.9997	.9999	1.000	1.000
430	.0023	.0178	.0671	.1688	.3214	.4996	.6688	.8034	.8950	.9494	.9778	.9912	.9968	.9989	.9997	.9999	1.000	1.000
440	.0023	.0174	.0660	.1665	.3179	.4955	.6649	.8003	.8929	.9481	.9772	.9908	.9966	.9989	.9996	.9999	1.000	1.000
450	.0022	.0171	.0649	.1643	.3145	.4916	.6611	.7972	.8908	.9469	.9765	.9905	.9965	.9988	.9996	.9999	1.000	1.000
460	.0022	.0168	.0639	.1621	.3112	.4877	.6574	.7942	.8887	.9456	.9759	.9902	.9964	.9988	.9996	.9999	1.000	1.000
470	.0021	.0164	.0629	.1600	.3081	.4839	.6538	.7913	.8867	.9444	.9752	.9899	.9962	.9987	.9996	.9999	1.000	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
480	.0021	.0161	.0619	.1580	.3050	.4802	.6502	.7884	.8847	.9432	.9745	.9896	.9961	.9987	.9996	.9999	1.000	1.000
490	.0020	.0159	.0610	.1560	.3019	.4766	.6467	.7855	.8827	.9420	.9739	.9893	.9960	.9986	.9995	.9999	1.000	1.000
500	.0020	.0156	.0601	.1541	.2990	.4731	.6433	.7827	.8807	.9408	.9732	.9890	.9958	.9985	.9995	.9999	1.000	1.000
510	.0020	.0153	.0592	.1522	.2961	.4696	.6399	.7800	.8788	.9396	.9726	.9887	.9957	.9985	.9995	.9999	1.000	1.000
520	.0019	.0151	.0583	.1504	.2934	.4663	.6366	.7772	.8769	.9384	.9720	.9883	.9956	.9984	.9995	.9998	1.000	1.000
530	.0019	.0148	.0575	.1487	.2906	.4630	.6333	.7746	.8749	.9372	.9713	.9880	.9954	.9984	.9995	.9998	1.000	1.000
540	.0019	.0146	.0567	.1470	.2880	.4598	.6302	.7719	.8731	.9360	.9707	.9877	.9953	.9983	.9994	.9998	1.000	1.000
550	.0018	.0143	.0560	.1453	.2854	.4566	.6270	.7693	.8712	.9349	.9700	.9874	.9951	.9983	.9994	.9998	.9999	1.000
560	.0018	.0141	.0552	.1437	.2829	.4536	.6240	.7667	.8694	.9337	.9694	.9871	.9950	.9982	.9994	.9998	.9999	1.000
570	.0018	.0139	.0545	.1422	.2805	.4505	.6210	.7642	.8675	.9326	.9688	.9868	.9949	.9982	.9994	.9998	.9999	1.000
580	.0017	.0137	.0538	.1407	.2781	.4476	.6180	.7617	.8657	.9314	.9681	.9864	.9947	.9981	.9994	.9998	.9999	1.000
590	.0017	.0135	.0531	.1392	.2757	.4447	.6151	.7593	.8640	.9303	.9675	.9861	.9946	.9980	.9993	.9998	.9999	1.000
600	.0017	.0133	.0524	.1378	.2734	.4419	.6122	.7568	.8622	.9292	.9669	.9858	.9944	.9980	.9993	.9998	.9999	1.000
610	.0016	.0131	.0518	.1363	.2712	.4391	.6094	.7545	.8605	.9281	.9662	.9855	.9943	.9979	.9993	.9998	.9999	1.000
620	.0016	.0129	.0512	.1350	.2690	.4364	.6066	.7521	.8587	.9270	.9656	.9852	.9941	.9979	.9993	.9998	.9999	1.000
630	.0016	.0127	.0506	.1336	.2669	.4337	.6039	.7498	.8570	.9259	.9650	.9849	.9940	.9978	.9993	.9998	.9999	1.000
640	.0016	.0126	.0500	.1323	.2648	.4311	.6012	.7475	.8553	.9248	.9644	.9846	.9938	.9977	.9992	.9998	.9999	1.000
650	.0015	.0124	.0494	.1311	.2627	.4285	.5986	.7452	.8537	.9237	.9638	.9842	.9937	.9977	.9992	.9997	.9999	1.000
660	.0015	.0122	.0488	.1298	.2607	.4260	.5960	.7430	.8520	.9226	.9631	.9839	.9936	.9976	.9992	.9997	.9999	1.000
670	.0015	.0121	.0483	.1286	.2588	.4235	.5935	.7408	.8504	.9216	.9625	.9836	.9934	.9976	.9992	.9997	.9999	1.000
680	.0015	.0119	.0478	.1274	.2569	.4211	.5909	.7386	.8487	.9205	.9619	.9833	.9933	.9975	.9991	.9997	.9999	1.000
690	.0014	.0118	.0472	.1263	.2550	.4187	.5885	.7364	.8471	.9195	.9613	.9830	.9931	.9974	.9991	.9997	.9999	1.000
700	.0014	.0116	.0467	.1251	.2531	.4163	.5860	.7343	.8455	.9184	.9607	.9827	.9930	.9974	.9991	.9997	.9999	1.000

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
710	.0014	.0115	.0462	.1240	.2513	.4140	.5836	.7322	.8439	.9174	.9601	.9824	.9928	.9973	.9991	.9997	.9999	1.000
720	.0014	.0113	.0457	.1230	.2495	.4117	.5812	.7301	.8424	.9164	.9595	.9820	.9927	.9972	.9990	.9997	.9999	1.000
730	.0014	.0112	.0453	.1219	.2478	.4095	.5789	.7280	.8408	.9153	.9589	.9817	.9925	.9972	.9990	.9997	.9999	1.000
740	.0014	.0111	.0448	.1209	.2461	.4073	.5766	.7260	.8393	.9143	.9583	.9814	.9924	.9971	.9990	.9997	.9999	1.000
750	.0013	.0109	.0444	.1198	.2444	.4052	.5743	.7240	.8378	.9133	.9577	.9811	.9922	.9971	.9990	.9997	.9999	1.000
760	.0013	.0108	.0439	.1188	.2428	.4030	.5721	.7220	.8363	.9123	.9571	.9808	.9921	.9970	.9989	.9996	.9999	1.000
770	.0013	.0107	.0435	.1179	.2412	.4009	.5699	.7201	.8348	.9113	.9565	.9805	.9919	.9969	.9989	.9996	.9999	1.000
780	.0013	.0106	.0431	.1169	.2396	.3989	.5677	.7181	.8333	.9103	.9560	.9802	.9918	.9969	.9989	.9996	.9999	1.000
790	.0013	.0104	.0427	.1160	.2380	.3969	.5656	.7162	.8318	.9093	.9554	.9799	.9916	.9968	.9989	.9996	.9999	1.000
800	.0012	.0103	.0423	.1151	.2365	.3949	.5634	.7143	.8304	.9084	.9548	.9796	.9915	.9967	.9988	.9996	.9999	1.000
810	.0012	.0102	.0419	.1142	.2350	.3929	.5614	.7124	.8289	.9074	.9542	.9793	.9913	.9967	.9988	.9996	.9999	1.000
820	.0012	.0101	.0415	.1133	.2335	.3910	.5593	.7106	.8275	.9064	.9536	.9789	.9912	.9966	.9988	.9996	.9999	1.000
830	.0012	.0100	.0411	.1124	.2320	.3891	.5572	.7088	.8261	.9055	.9531	.9786	.9910	.9965	.9988	.9996	.9999	1.000
840	.0012	.0099	.0407	.1116	.2306	.3872	.5552	.7069	.8247	.9045	.9525	.9783	.9909	.9965	.9987	.9996	.9999	1.000
850	.0012	.0098	.0404	.1107	.2292	.3853	.5533	.7051	.8233	.9036	.9519	.9780	.9907	.9964	.9987	.9996	.9999	1.000
860	.0012	.0097	.0400	.1099	.2278	.3835	.5513	.7034	.8219	.9026	.9514	.9777	.9906	.9963	.9987	.9996	.9999	1.000
870	.0011	.0096	.0397	.1091	.2265	.3817	.5494	.7016	.8205	.9017	.9508	.9774	.9904	.9963	.9986	.9995	.9999	1.000
880	.0011	.0095	.0393	.1083	.2251	.3799	.5474	.6999	.8192	.9008	.9502	.9771	.9903	.9962	.9986	.9995	.9999	1.000
890	.0011	.0094	.0390	.1075	.2238	.3782	.5455	.6982	.8178	.8998	.9497	.9768	.9901	.9961	.9986	.9995	.9998	1.000
900	.0011	.0093	.0386	.1068	.2225	.3765	.5437	.6964	.8165	.8989	.9491	.9765	.9900	.9961	.9986	.9995	.9998	1.000
910	.0011	.0092	.0383	.1060	.2212	.3748	.5418	.6948	.8151	.8980	.9486	.9762	.9898	.9960	.9985	.9995	.9998	1.000
920	.0011	.0091	.0380	.1053	.2200	.3731	.5400	.6931	.8138	.8971	.9480	.9759	.9897	.9959	.9985	.9995	.9998	1.000
930	.0011	.0090	.0377	.1045	.2187	.3714	.5382	.6914	.8125	.8962	.9475	.9756	.9895	.9959	.9985	.9995	.9998	.9999

Table 1 (contd)

n	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8	k=9	k=10	k=11	k=12	k=13	k=14	k=15	k=16	k=17	k=18
940	.0011	.0090	.0374	.1038	.2175	.3698	.5364	.6898	.8112	.8953	.9469	.9753	.9894	.9958	.9984	.9995	.9998	.9999
950	.0011	.0089	.0371	.1031	.2163	.3682	.5347	.6882	.8099	.8944	.9464	.9750	.9892	.9957	.9984	.9995	.9998	.9999
960	.0010	.0088	.0368	.1024	.2151	.3666	.5329	.6866	.8087	.8935	.9458	.9747	.9891	.9957	.9984	.9994	.9998	.9999
970	.0010	.0087	.0365	.1018	.2140	.3651	.5312	.6850	.8074	.8926	.9453	.9744	.9889	.9956	.9984	.9994	.9998	.9999
980	.0010	.0086	.0362	.1011	.2128	.3635	.5295	.6834	.8061	.8918	.9447	.9741	.9888	.9955	.9983	.9994	.9998	.9999
990	.0010	.0086	.0359	.1004	.2117	.3620	.5278	.6818	.8049	.8909	.9442	.9738	.9886	.9955	.9983	.9994	.9998	.9999
1000	.0010	.0085	.0357	.0998	.2106	.3605	.5261	.6803	.8036	.8900	.9436	.9735	.9885	.9954	.9983	.9994	.9998	.9999

Source: The author's calculations.

Table 2

Interpolated quantiles of Goldfeld-Quandt peak test

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
5	3.9800	3.8600	3.5000	2.9714	2.6286
6	4.5867	4.1067	3.7647	3.3412	2.8089
7	4.8377	4.5497	3.9252	3.5823	2.9532
8	4.9734	4.7677	4.1505	3.7458	3.1501
9	5.3285	4.9047	4.4198	3.8704	3.3311
10	5.5903	5.0168	4.5997	3.9725	3.4710
11	5.7540	5.3115	4.7318	4.1477	3.5853
12	5.8680	5.5089	4.8352	4.3145	3.6823
13	5.9537	5.6515	4.9200	4.4463	3.7670
14	6.0916	5.7604	4.9921	4.5544	3.8426
15	6.3069	5.8473	5.1549	4.6456	3.9111
16	6.4671	5.9190	5.2973	4.7244	3.9740
17	6.5909	5.9798	5.4141	4.7938	4.0584
18	6.6897	6.1089	5.5120	4.8557	4.1516
19	6.7707	6.2537	5.5958	4.9118	4.2342
20	6.8385	6.3730	5.6686	4.9631	4.3080
21	6.8965	6.4730	5.7328	5.0232	4.3748
22	6.9467	6.5583	5.7900	5.1181	4.4356
23	6.9909	6.6320	5.8416	5.2022	4.4916
24	7.1062	6.6965	5.8884	5.2775	4.5433
25	7.2240	6.7536	5.9314	5.3455	4.5915
26	7.3249	6.8046	5.9710	5.4072	4.6365
27	7.4124	6.8505	6.0197	5.4638	4.6788
28	7.4889	6.8922	6.1048	5.5158	4.7188
29	7.5564	6.9302	6.1814	5.5639	4.7566
30	7.6165	6.9652	6.2509	5.6087	4.7925
31	7.6704	6.9976	6.3141	5.6505	4.8268
32	7.7190	7.0808	6.3721	5.6896	4.8595
33	7.7631	7.1600	6.4254	5.7265	4.8909
34	7.8033	7.2317	6.4747	5.7613	4.9210
35	7.8402	7.2969	6.5204	5.7942	4.9500
36	7.8742	7.3565	6.5630	5.8255	4.9779
37	7.9057	7.4112	6.6028	5.8553	5.0082

Table 2 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
38	7.9349	7.4616	6.6401	5.8836	5.0515
39	7.9621	7.5082	6.6752	5.9108	5.0925
40	7.9876	7.5514	6.7083	5.9368	5.1315
41	8.0269	7.5917	6.7396	5.9618	5.1685
42	8.0778	7.6293	6.7693	5.9858	5.2039
43	8.1242	7.6646	6.7974	6.0181	5.2377
44	8.1666	7.6977	6.8242	6.0625	5.2700
45	8.2055	7.7288	6.8498	6.1047	5.3010
46	8.2414	7.7582	6.8742	6.1447	5.3308
47	8.2744	7.7860	6.8976	6.1827	5.3594
48	8.3049	7.8123	6.9200	6.2190	5.3870
49	8.3332	7.8373	6.9415	6.2535	5.4135
50	8.3595	7.8611	6.9622	6.2866	5.4392
51	8.3839	7.8838	6.9821	6.3182	5.4639
52	8.4067	7.9054	7.0031	6.3485	5.4879
53	8.4280	7.9261	7.0455	6.3776	5.5111
54	8.4479	7.9459	7.0857	6.4055	5.5336
55	8.4666	7.9649	7.1241	6.4324	5.5554
56	8.4841	7.9831	7.1607	6.4583	5.5766
57	8.5006	8.0012	7.1956	6.4832	5.5972
58	8.5161	8.0323	7.2291	6.5073	5.6173
59	8.5308	8.0616	7.2611	6.5305	5.6368
60	8.5446	8.0892	7.2918	6.5530	5.6558

Source: The author's calculations.

3) if $G = g(n, \alpha) - 1$ H_0 is rejected with the probability

$$p_{rand}(n, \alpha) = \frac{\alpha - p(G \geq g(n, \alpha))}{p(G = g(n, \alpha) - 1)} \tag{12}$$

We call this procedure *the randomized peak test*. Its size is obviously equal to α.

The *interpolated critical values*

$$g_i(n, \alpha) = g(n, \alpha) + p_{rand}(n, \alpha) \tag{13}$$

(cf. Tomaszewicz, 1985, p. 155 and also Domański, 1986, p. 123) for $\alpha = 0.10$, 0.05, 0.01 and $n = 5, 6, \ldots, 100$ are given in Table 2. Using tables in computer programs is inconvenient. It is much easier to use an approximation. The coefficients $\lambda_j(\alpha)$ of the approximation given in the form

$$g_i(n, \alpha) \approx$$

$$\approx \hat{g}_i(n, \alpha) = \lambda_{-2}(\alpha)/n^2 + \lambda_{-1}(\alpha)/n + \lambda_0(\alpha) + \lambda_1(\alpha)n + \lambda_2(\alpha)n^2 \tag{14}$$

for some most frequently applied significance levels are to be found in Table 3.

Table 3

The $\lambda_j(\alpha)$ coefficients

0.01000	17.76	-16.902	6.3512	0.07571	-0.000579
0.02000	20.50	-15.885	5.8502	0.07162	-0.000504
0.05000	38.12	-19.644	5.6771	0.04767	-0.000272
0.10000	25.76	-16.159	5.0263	0.04025	-0.000176
0.20000	32.29	-16.293	4.4349	0.03225	-0.000122

Source: The author's calculations.

Each approximation (14) corresponds to the approximation of critical value $g(n, \alpha)$:

$$\hat{g}(n, \alpha) = -\text{ entier } (-\hat{g}_i(n, \alpha)) + 1,$$

the approximation of randomization probability

$$\hat{p}_{rand}(n, \alpha) = \hat{g}(n, \alpha) - \hat{g}_i(n, \alpha)$$

the size of the test

$$\hat{\alpha}(n, \alpha) = P(G \geqslant g(n, \alpha)) + P_{rand}(n, \alpha)P(G = g(n, \alpha) - 1),$$

which obviously, is close to α for a good approximation.

The approximation errors

$$\hat{g}_i(n, \alpha) - g_i(n, \alpha)$$

and randomized test size errors

$$\delta(n, \alpha) = \hat{\alpha}(n, \alpha) - \alpha$$

are given in Table 4 and 5, respectively. It can be seen that

Table 4

Differences between interpolated quantiles of Goldfeld-Quandt peak test and their approximations (14)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
5	0.0651	-0.0214	0.0046	0.0502	-0.0027
6	-0.1259	0.0769	-0.0265	-0.0574	-0.0036
7	-0.0372	-0.0739	0.0440	-0.0657	0.0328
8	0.1111	-0.0422	0.0307	-0.0262	0.0028
9	-0.0017	0.0372	-0.0477	0.0264	-0.0276
10	-0.0526	0.1156	-0.0563	0.0803	-0.0323
11	-0.0300	-0.0093	-0.0339	0.0439	-0.0248
12	0.0231	-0.0534	0.0026	0.0017	-0.0116
13	0.0887	-0.0562	0.0454	-0.0172	0.0042
14	0.0893	-0.0365	0.0905	-0.0220	0.0208
15	0.0017	-0.0042	0.0360	-0.0181	0.0373
16	-0.0399	0.0352	-0.0058	-0.0086	0.0534
17	-0.0529	0.0786	-0.0286	0.0044	0.0428
18	-0.0476	0.0477	-0.0385	0.0197	0.0186
19	-0.0305	-0.0042	-0.0392	0.0364	0.0013
20	-0.0056	-0.0352	-0.0336	0.0541	-0.0109
21	0.0246	-0.0513	-0.0233	0.0595	-0.0192
22	0.0582	-0.0564	-0.0097	0.0271	-0.0243
23	0.0939	-0.0534	0.0063	0.0027	-0.0270
24	0.0549	-0.0444	0.0241	-0.0154	-0.0277
25	0.0103	-0.0308	0.0431	-0.0283	-0.0268
26	-0.0206	-0.0140	0.0629	-0.0371	-0.0247
27	-0.0409	0.0054	0.0714	-0.0425	-0.0215
28	-0.0529	0.0267	0.0413	-0.0450	-0.0174
29	-0.0585	0.0493	0.0177	-0.0453	-0.0127
30	-0.0590	0.0728	-0.0004	-0.0436	-0.0074
31	-0.0556	0.0969	-0.0142	-0.0404	-0.0016
32	-0.0490	0.0682	-0.0242	-0.0358	0.0046
33	-0.0401	0.0417	-0.0311	-0.0300	0.0111
34	-0.0293	0.0209	-0.0354	-0.0233	0.0178
35	-0.0171	0.0049	-0.0376	-0.0159	0.0247
36	-0.0039	-0.0072	-0.0379	-0.0077	0.0319

Table 4 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
37	0.0100	-0.0160	-0.0367	0.0010	0.0358
38	0.0244	-0.0221	-0.0342	0.0101	0.0259
39	0.0391	-0.0259	-0.0306	0.0196	0.0176
40	0.0538	-0.0279	-0.0261	0.0295	0.0106
41	0.0531	-0.0283	-0.0209	0.0395	0.0049
42	0.0392	-0.0276	-0.0151	0.0498	0.0002
43	0.0282	-0.0258	-0.0088	0.0511	-0.0036
44	0.0196	-0.0232	-0.0021	0.0394	-0.0065
45	0.0129	-0.0201	0.0049	0.0294	-0.0086
46	0.0079	-0.0165	0.0121	0.0209	-0.0101
47	0.0042	-0.0127	0.0195	0.0137	-0.0109
48	0.0015	-0.0086	0.0269	0.0076	-0.0112
49	-0.0004	-0.0045	0.0344	0.0026	-0.0111
50	-0.0017	-0.0005	0.0419	-0.0015	-0.0104
51	-0.0026	0.0034	0.0493	-0.0047	-0.0094
52	-0.0032	0.0071	0.0549	-0.0072	-0.0081
53	-0.0037	0.0106	0.0383	-0.0090	-0.0064
54	-0.0042	0.0137	0.0230	-0.0103	-0.0045
55	-0.0048	0.0165	0.0089	-0.0110	-0.0023
56	-0.0056	0.0188	-0.0043	-0.0112	0.0001
57	-0.0068	0.0201	-0.0165	-0.0111	0.0027
58	-0.0083	0.0072	-0.0280	-0.0106	0.0054
59	-0.0102	-0.0050	-0.0387	-0.0097	0.0083
60	-0.0126	-0.0168	-0.0489	-0.0086	0.0113

Source: The author's calculations.

Table 5

Test size errors for the approximation (14)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
5	-0.00204	0.00179	-0.00039	-0.01014	0.00078
6	0.00262	-0.00160	0.00312	0.00678	0.00112
7	0.00129	0.00257	-0.00642	0.00958	-0.01056
8	-0.00197	0.00205	-0.00149	0.00440	-0.00047

Table 5 (contd)

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
9	0.00002	-0.00230	0.00295	-0.00489	0.00511
10	0.00092	-0.00202	0.00417	-0.00941	0.00644
11	0.00068	0.00021	0.00290	-0.00375	0.00523
12	-0.00064	0.00149	-0.00025	-0.00016	0.00255
13	-0.00184	0.00186	-0.00479	0.00181	-0.00095
14	-0.00081	0.00140	-0.00406	0.00251	-0.00484
15	-0.00002	0.00018	-0.00156	0.00221	-0.00888
16	0.00052	-0.00170	0.00028	0.00112	-0.00983
17	0.00080	-0.00195	0.00152	-0.00059	-0.00581
18	0.00082	-0.00082	0.00222	-0.00279	-0.00265
19	0.00059	0.00008	0.00244	-0.00537	-0.00019
20	0.00012	0.00076	0.00223	-0.00677	0.00167
21	-0.00058	0.00121	0.00164	-0.00419	0.00302
22	-0.00141	0.00145	0.00072	-0.00201	0.00393
23	-0.00091	0.00149	-0.00049	-0.00021	0.00446
24	-0.00047	0.00133	-0.00197	0.00126	0.00468
25	-0.00010	0.00099	-0.00368	0.00242	0.00463
26	0.00021	0.00048	-0.00373	0.00329	0.00433
27	0.00046	-0.00020	-0.00258	0.00390	0.00383
28	0.00064	-0.00102	-0.00157	0.00428	0.00316
29	0.00076	-0.00197	-0.00071	0.00444	0.00233
30	0.00082	-0.00199	0.00002	0.00440	0.00137
31	0.00082	-0.00150	0.00062	0.00418	0.00030
32	0.00077	-0.00107	0.00111	0.00380	-0.00087
33	0.00066	-0.00069	0.00148	0.00327	-0.00212
34	0.00051	-0.00037	0.00175	0.00260	-0.00345
35	0.00031	-0.00009	0.00192	0.00180	-0.00484
36	0.00008	0.00014	0.00201	0.00090	-0.00548
37	-0.00020	0.00032	0.00200	-0.00011	-0.00422
38	-0.00052	0.00047	0.00192	-0.00122	-0.00312
39	-0.00084	0.00057	0.00177	-0.00240	-0.00215
40	-0.00058	0.00064	0.00156	-0.00366	-0.00132
41	-0.00034	0.00068	0.00128	-0.00490	-0.00061

Table 5 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
42	-0.00010	0.00068	0.00095	-0.00405	-0.00002
43	0.00014	0.00066	0.00056	-0.00328	0.00047
44	0.00036	0.00061	0.00014	-0.00259	0.00085
45	0.00058	0.00055	-0.00033	-0.00197	0.00115
46	0.00079	0.00047	-0.00083	-0.00143	0.00136
47	0.00100	0.00037	-0.00136	-0.00096	0.00149
48	0.00120	0.00026	-0.00192	-0.00054	0.00155
49	0.00141	0.00014	-0.00250	-0.00019	0.00154
50	0.00161	0.00002	-0.00293	0.00011	0.00147
51	0.00181	-0.00011	-0.00236	0.00035	0.00134
52	0.00202	-0.00024	-0.00183	0.00055	0.00116
53	0.00223	-0.00036	-0.00130	0.00070	0.00093
54	0.00244	-0.00048	-0.00080	0.00081	0.00065
55	0.00266	-0.00059	-0.00032	0.00088	0.00034
56	0.00288	-0.00064	0.00016	0.00092	-0.00002
57	0.00311	-0.00027	0.00062	0.00092	-0.00040
58	0.00336	0.00010	0.00106	0.00089	-0.00081
59	0.00361	0.00048	0.00150	0.00083	-0.00125
60	0.00388	0.00086	0.00193	0.00074	-0.00172

Source: The author's calculations.

- except very small sample sizes where the approximation error is quite large - the accuracy of our approximation seems to be good enough for practical statistical research (2 promille for $\alpha = 5\%$ and 3-6 promille for $\alpha = 10\%$). However, this approximation can hardly be assumed excellent and it is not to be recommended for analyses when high precision is needed.

The tables presented here as well as the approximation formulae (which turned out to be rather exact) seem to eliminate some barriers to the application of Goldfeld-Quandt test in its classical or randomized form. This applies to the case where residuals $\{e_t\}$ are independent and have the same distribution.

3. THE PEAK TEST FOR THE LINEAR TREND MODEL

It is well known, however that the distribution of e_t residuals is not spherical, so the probability distribution function of the number of peaks depends on the matrix X and thus the application of either Goldfeld-Quandt tables or the Tables 6-9 we proposed may raise some doubts. Making comments on that Goldfeld and Quandt (1967) write that the differences between the exact probability distribution function and the one obtained under the assumption of independence of e_t residuals are the largest for small numbers of peaks and relatively quite small for large ones. Hence, considering that the critical region is mainly right-hand sided, the error resulting from the application of critical values from the tables mentioned above is on the whole not large. Because the dependencies between e_t residuals diminish as the sample size grows it is assumed (we lack any formal evidence as yet)[2] that distribution (9) is the limit for the distribution of statistics G and that it can be applied in practise starting from $n \geqslant 15$.

Table 6

Estimates of the interpolated quantiles of the peak test statistic for the linear trend

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
5	3.9491	3.8360	3.4966	2.9820	2.6872
6	4.5385	4.0256	3.7995	3.4596	2.9080
7	4.8792	4.6377	3.9789	3.6856	3.0991
8	5.2137	4.8718	4.3590	3.8497	3.3316
9	5.4312	4.9571	4.5422	3.9479	3.4653
10	5.7357	5.2952	4.7294	4.1487	3.6205
11	5.8767	5.5342	4.8547	4.3502	3.7242
12	5.9198	5.6111	4.9098	4.4677	3.8020
13	6.1900	5.7928	5.0256	4.5980	3.8868

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[2] In our opinion this proof cannot be made without some assumptions concerning random term distribution.

Table 6 (contd)

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
14	6.3445	5.8618	5.1593	4.6574	3.9318
15	6.4597	5.9142	5.3154	4.7467	4.0059
16	6.6048	5.9539	5.4004	4.8007	4.0965
17	6.7287	6.1968	5.5799	4.9073	4.2307
18	6.7385	6.2798	5.6379	4.9478	4.2868
19	6.8940	6.4332	5.6921	4.9770	4.3559
20	6.8468	6.4435	5.7497	5.0909	4.4232
21	6.9234	6.5584	5.8068	5.1498	4.4726
22	7.1266	6.6774	5.8675	5.2709	4.5474
23	7.0488	6.6863	5.9004	5.3473	4.6104
24	7.3398	6.7703	5.9120	5.3755	4.6344
25	7.3143	6.8047	5.9729	5.4086	4.6548
26	7.4884	6.9073	6.1554	5.5498	4.7462
27	7.5899	6.9541	6.1888	5.5761	4.7743
28	7.4922	6.9046	6.1314	5.5510	4.7694
29	7.6713	6.9906	6.2864	5.6379	4.8164
30	7.7852	7.1141	6.3509	5.6793	4.8648
31	7.7152	7.0823	6.4178	5.7335	4.9032
32	7.7836	7.1988	6.5207	5.8121	4.9550
33	7.9799	7.4774	6.6346	5.8719	4.9901
34	7.8656	7.3280	6.5942	5.8660	4.9845
35	7.9109	7.4158	6.6217	5.8772	5.0305
36	7.8756	7.4147	6.6429	5.9016	5.0952
37	7.9912	7.5526	6.7260	5.9697	5.1927
38	7.9874	7.5672	6.7074	5.9198	5.1262
39	8.0541	7.5862	6.7363	5.9651	5.1338
40	8.1852	7.6559	6.7857	6.0093	5.2173
41	8.2000	7.6587	6.7919	6.0155	5.2332
42	8.2990	7.7126	6.8171	6.0796	5.3080
43	8.3333	7.7606	6.8784	6.1548	5.3214
44	8.2234	7.7516	6.9088	6.1614	5.3936
45	8.3297	7.7674	6.9055	6.2205	5.3938
46	8.3488	7.7561	6.8847	6.1903	5.4032
47	8.5360	7.8956	6.9763	6.2799	5.4341
48	8.5122	7.8833	6.9724	6.2818	5.4555

Table 6 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
49	8.6000	7.9017	7.0347	6.3333	5.4631
50	8.5378	7.8938	7.0088	6.3725	5.5259
51	8.6115	7.9538	7.0308	6.3645	5.5052
52	8.7246	8.0000	7.1525	6.4170	5.5433
53	8.5514	7.8835	7.0312	6.3903	5.5349
54	8.6014	7.9620	7.1467	6.4333	5.5698
55	8.7424	7.9948	7.2094	6.4815	5.5987
56	8.8908	8.0504	7.2181	6.5217	5.6461
57	8.7234	8.0142	7.2199	6.5294	5.6616
58	8.6641	7.9663	7.1891	6.5308	5.6848
59	8.8054	8.1342	7.3381	6.5882	5.6885
60	8.8710	8.2258	7.3645	6.6209	5.7334

Source: The author's calculations.

Table 7

The $\gamma_j(\alpha)$ coefficients

α	$\gamma_{-2}(\alpha)$	$\gamma_{-1}(\alpha)$	$\gamma_0(\alpha)$	$\gamma_1(\alpha)$	$\gamma_2(\alpha)$
0.01000	2.93	-14.968	6.5262	0.06605	-0.000391
0.02000	3.29	-12.596	5.7832	0.07604	-0.000566
0.05000	22.04	-16.027	5.5490	0.05400	-0.000348
0.10000	10.50	-12.526	4.8981	0.04635	-0.000247
0.20000	17.34	-12.437	4.2889	0.03835	-0.000186

Source: The author's calculations.

Table 8

Differences between interpolated quantiles of Goldfeld-Quandt peak test and the statistic for linear trend

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
5	0.0098	0.0983	0.0137	-0.0670	-0.0534
6	0.0915	-0.1045	-0.0367	-0.0301	-0.1121
7	-0.0532	-0.0058	-0.1449	-0.0530	-0.1721

Table 8 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
8	-0.2310	-0.0646	-0.1492	-0.1056	-0.1499
9	-0.1335	-0.1580	-0.0783	-0.1627	-0.1200
10	-0.0898	-0.2435	-0.0721	-0.2168	-0.1123
11	-0.1149	-0.1217	-0.0943	-0.1785	-0.1154
12	-0.1675	-0.0784	-0.1292	-0.1334	-0.1238
13	-0.2311	-0.0751	-0.1697	-0.1112	-0.1347
14	-0.2286	-0.0935	-0.2124	-0.1031	-0.1467
15	-0.1373	-0.1241	-0.1553	-0.1038	-0.1590
16	-0.0918	-0.1616	-0.1111	-0.1102	-0.1711
17	-0.0749	-0.2029	-0.0859	-0.1205	-0.1570
18	-0.0763	-0.1700	-0.0739	-0.1332	-0.1297
19	-0.0898	-0.1160	-0.0712	-0.1476	-0.1095
20	-0.1113	-0.0830	-0.0750	-0.1632	-0.0948
21	-0.1384	-0.0650	-0.0836	-0.1667	-0.0843
22	-0.1692	-0.0580	-0.0957	-0.1326	-0.0772
23	-0.2025	-0.0592	-0.1103	-0.1066	-0.0727
24	-0.1616	-0.0665	-0.1268	-0.0871	-0.0704
25	-0.1153	-0.0782	-0.1445	-0.0728	-0.0698
26	-0.0831	-0.0934	-0.1632	-0.0628	-0.0707
27	-0.0620	-0.1111	-0.1705	-0.0563	-0.0727
28	-0.0496	-0.1307	-0.1393	-0.0526	-0.0756
29	-0.0440	-0.1517	-0.1147	-0.0512	-0.0794
30	-0.0439	-0.1735	-0.0954	-0.0518	-0.0837
31	-0.0481	-0.1960	-0.0806	-0.0541	-0.0886
32	-0.0559	-0.1656	-0.0695	-0.0576	-0.0939
33	-0.0664	-0.1374	-0.0615	-0.0623	-0.0995
34	-0.0792	-0.1149	-0.0561	-0.0679	-0.1054
35	-0.0939	-0.0971	-0.0527	-0.0743	-0.1115
36	-0.1099	-0.0832	-0.0512	-0.0814	-0.1178
37	-0.1271	-0.0726	-0.0511	-0.0889	-0.1208
38	-0.1452	-0.0646	-0.0523	-0.0968	-0.1101
39	-0.1639	-0.0588	-0.0545	-0.1051	-0.1009
40	-0.1830	-0.0549	-0.0575	-0.1136	-0.0930
41	-0.1873	-0.0523	-0.0611	-0.1223	-0.0862
42	-0.1786	-0.0510	-0.0653	-0.1311	-0.0804

Table 8 (contd)

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
43	-0.1732	-0.0506	-0.0699	-0.1309	-0.0756
44	-0.1707	-0.0509	-0.0748	-0.1176	-0.0715
45	-0.1705	-0.0517	-0.0798	-0.1059	-0.0682
46	-0.1724	-0.0528	-0.0851	-0.0956	-0.0654
47	-0.1759	-0.0542	-0.0903	-0.0865	-0.0632
48	-0.1809	-0.0557	-0.0956	-0.0785	-0.0615
49	-0.1870	-0.0571	-0.1008	-0.0715	-0.0602
50	-0.1942	-0.0584	-0.1058	-0.0653	-0.0592
51	-0.2022	-0.0595	-0.1107	-0.0598	-0.0586
52	-0.2108	-0.0603	-0.1137	-0.0550	-0.0582
53	-0.2199	-0.0608	-0.0943	-0.0508	-0.0580
54	-0.2294	-0.0608	-0.0762	-0.0470	-0.0580
55	-0.2392	-0.0603	-0.0590	-0.0436	-0.0582
56	-0.2491	-0.0594	-0.0428	-0.0406	-0.0585
57	-0.2592	-0.0573	-0.0273	-0.0379	-0.0589
58	-0.2693	-0.0409	-0.0125	-0.0355	-0.0594
59	-0.2793	-0.0251	0.0017	-0.0332	-0.0599
60	-0.2892	-0.0097	0.0154	-0.0311	-0.0604

Source: The author's calculations.

Table 9

Estimates of test size error when using Goldfeld-Quandt statistic for the linear trend case

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
5	-0.0027	-0.0021	-0.0003	0.0036	0.0199
6	-0.0009	-0.0016	0.0051	0.0174	0.0349
7	0.0017	0.0036	0.0092	0.0176	0.0336
8	0.0041	0.0061	0.0122	0.0201	0.0350
9	0.0016	0.0038	0.0088	0.0161	0.0278
10	0.0033	0.0063	0.0112	0.0191	0.0344
11	0.0036	0.0065	0.0122	0.0201	0.0329
12	0.0017	0.0033	0.0084	0.0173	0.0285
13	0.0037	0.0055	0.0107	0.0185	0.0287

Table 9 (contd)

n	α = 0.01	α = 0.02	α = 0.05	α = 0.10	α = 0.20
14	0.0030	0.0043	0.0078	0.0130	0.0223
15	0.0019	0.0034	0.0080	0.0136	0.0228
16	0.0017	0.0019	0.0056	0.0108	0.0203
17	0.0026	0.0050	0.0104	0.0168	0.0255
18	0.0011	0.0037	0.0083	0.0139	0.0205
19	0.0027	0.0039	0.0064	0.0105	0.0196
20	0.0002	0.0017	0.0062	0.0129	0.0186
21	0.0007	0.0023	0.0056	0.0096	0.0164
22	0.0025	0.0033	0.0065	0.0128	0.0191
23	0.0007	0.0017	0.0053	0.0131	0.0209
24	0.0024	0.0021	0.0022	0.0091	0.0162
25	0.0009	0.0018	0.0037	0.0056	0.0117
26	0.0021	0.0041	0.0090	0.0139	0.0201
27	0.0025	0.0041	0.0066	0.0113	0.0178
28	0.0000	0.0005	0.0010	0.0035	0.0099
29	0.0016	0.0026	0.0045	0.0077	0.0109
30	0.0025	0.0032	0.0044	0.0076	0.0142
31	0.0007	0.0014	0.0051	0.0092	0.0151
32	0.0011	0.0020	0.0082	0.0138	0.0187
33	0.0043	0.0063	0.0117	0.0163	0.0199
34	0.0012	0.0018	0.0070	0.0117	0.0127
35	0.0014	0.0024	0.0058	0.0098	0.0136
36	0.0000	0.0013	0.0047	0.0094	0.0162
37	0.0020	0.0032	0.0078	0.0147	0.0237
38	0.0012	0.0025	0.0038	0.0046	0.0094
39	0.0013	0.0018	0.0038	0.0065	0.0050
40	0.0018	0.0026	0.0050	0.0086	0.0109
41	0.0012	0.0017	0.0034	0.0059	0.0083
42	0.0021	0.0021	0.0032	0.0073	0.0142
43	0.0020	0.0027	0.0056	0.0095	0.0110
44	0.0005	0.0017	0.0057	0.0066	0.0182
45	0.0011	0.0012	0.0041	0.0085	0.0128
46	0.0009	-0.0001	0.0008	0.0033	0.0105
47	0.0033	0.0035	0.0057	0.0070	0.0106
48	0.0025	0.0022	0.0038	0.0045	0.0100

Table 9 (contd)

n	$\alpha = 0.01$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.10$	$\alpha = 0.20$
49	0.0029	0.0022	0.0055	0.0058	0.0070
50	0.0021	0.0011	0.0033	0.0068	0.0129
51	0.0032	0.0023	0.0024	0.0036	0.0060
52	0.0044	0.0033	0.0053	0.0052	0.0083
53	0.0013	-0.0015	-0.0005	0.0010	0.0035
54	0.0022	0.0006	0.0022	0.0022	0.0056
55	0.0036	0.0011	0.0033	0.0040	0.0066
56	0.0048	0.0012	0.0022	0.0055	0.0107
57	0.0031	0.0002	0.0009	0.0041	0.0101
58	0.0019	-0.0017	-0.0015	0.0021	0.0111
59	0.0041	0.0011	0.0033	0.0050	0.0081
60	0.0051	0.0021	0.0030	0.0062	0.0125

Source: The author's calculations.

It is not recommended, however, to use the peak test for very small samples. Heyadat and Robson (1970) using a Monte Carlo experiment pointed out that the size of the test based on Goldfeld-Quandt tables considerably varies from the assumed significance level in case when, according to the authors' recommendations, it compares in (3) e_t residuals obtained by OLS. We aim at the evaluation of critical values of peak tests for the linear trend model case i.e. when matrix **X** in (1) has the following form

$$\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ 1 & n \end{bmatrix}.$$

The interpolated critical values were estimated by Monte-Carlo experiment[3] in which for every

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[3] A similar experiment performed in order to evaluate critical values of Geary's autocorrelation test is presented in Tomaszewicz (1984a).

$n = 5, 6, \ldots, 60$

$s = 10\ 000$ samples were drawn from the statistical space described by model (1) under the assumption that the distribution of random terms is spherical:

ε: $N(0, \sigma^2 I)$

In each sample residuals e_t were determined and this yielded the value of G statistics. On this way, for each n, series of $|e_t|$ was obtained; the number of events $G = g$ is denoted by s_g.

By substituting evaluations s_g/S to formulae (6)-(8) in place of probabilities $P(G = g)$ we obtained evaluations of total critical values $g(n, \alpha)$, randomizing probabilities $p_{rand}(n, \alpha)$ and interpolated critical values from some chosen significance levels.

On the basis of the observed interpolated quantiles the $\gamma(n, \alpha)$ coefficients of the rational function approximating these quantiles were determined.

$$\tilde{g}_i(n, \alpha) = \gamma_{-2}(\alpha)/n^2 + \gamma_{-1}(\alpha)/n + \gamma_0(\alpha) + \gamma_1(\alpha)n + \gamma_2(\alpha)n^2 \qquad (15)$$

Table 2 contains the evaluation of coefficients $\gamma_n(\alpha)$. The observed interpolated critical values, their approximations differences between them and the evaluations of differences in the size of test caused by the acception of the approximate value are presented in Table 3.

4. CONCLUSIONS

The differences between the observed (or approximated) interpolated quantiles and the ones calculated under the assumption of spherical distribution of e_t residuals are quite significant. The results included in Table 4 confirm that the latter are considerably smaller. Therefore, when applying them in practise one should take into account that the actual size of test will be larger than assumed.

The approximate quantiles we obtained are error weighted. Their estimation is fairly difficult as it consists of error of estimate $g(n, \alpha)$ and error of approximation. Evaluations of quantiles are in our opinion, exact enough to be used in practical research. However the problem of the evaluation of their accuracy should be studied.

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O TEŚCIE SZCZYTÓW GOLDFELDA-QUANDTA

Goldfeld i Quandt w 1965 r. zaproponowali nieparametryczny test homoskedastyczności składnika losowego w modelu ekonometrycznym. Statystyką tego testu jest liczba tzw. szczytów (peaks), które tworzą bezwzględne wartości reszt. Podali oni również rozkład liczby szczytów przy założeniu, że każda permutacja bezwzględnych wartości reszt jest jednakowo prawdopodobna. Tego warunku nie spełniają jednakże reszty regresji, stąd proponowany przez Goldfelda i Quandta rozkład nie jest rozkładem dokładnym.

W artykule zaprezentowano wyniki eksperymentu Monte-Carlo zmierzającego do oceny dokładnego rozkładu liczby szczytów oraz błędu, jaki popełnia się stosując rozkład Goldfelda-Quandta.